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Approximation algorithms for conflict-free channel assignment in wireless *ad hoc* networks

Peng-Jun Wan[1,2*,†], Chih-Wei Yi[1], Xiaohua Jia[2] and Dongsoo Kim[3]

[1]*Department of Computer Science, Illinois Institute of Technology, Chicago, IL 60616, U.S.A.*
[2]*Department of Computer Science, City University of Hong Kong, Kowloon, Hong Kong*
[3]*Department of Electrical And Computer Engineering, Indiana University Purdue University, West Lafayette, Indianapolis, IN 46202, U.S.A.*

Summary

Conflict-free channel assignment is a classic and fundamental problem in wireless *ad hoc* networks. It seeks an assignment of the fewest channels to a given set of radio nodes with specified transmission ranges without causing either primary collision or secondary collision. It is NP-hard even when all nodes are located in a plane and have the same transmission radii. We observe that a prior analysis of the approximation ratio of a classic greedy heuristic, FIRST-FIT in smallest-last ordering, is erroneous. In this paper, we provide a rigorous and tighter analysis of this heuristic and other greedy FIRST-FIT heuristics. We obtain an upper bound of 13 on the approximation ratios of both FIRST-FIT in smallest-last ordering and FIRST-FIT in radius-decreasing ordering. Such upper bound can be reduced to 12 if all nodes have quasi-uniform transmission radii. When all nodes have equal transmission radii, we obtain an upper bound of 7 on the approximation ratios of FIRST-FIT in smallest-last ordering, FIRST-FIT in distance-increasing ordering, and FIRST-FIT in lexicographic ordering. In addition, for nodes with equal transmission radii, we present a spatial divide-and-conquer heuristic with approximation ratios of 12. All these heuristics, except FIRST-FIT in smallest-last ordering, are modified to heuristics for maximum independent set with the same approximation ratios. Copyright © 2006 John Wiley & Sons, Ltd.

KEY WORDS: primary interference; secondary inteference; channel assignment; approximation algorithm

1. Introduction

A wireless *ad hoc* network is a collection of radio nodes (transceivers) located in a geographic region. Each node is equipped with an omnidirectional antenna and has limited transmission power. A communication session is established either through a single-hop radio transmission if the communication parties are close enough, or through relaying by intermediate nodes otherwise. A channel assignment to the nodes in a wireless *ad hoc* network should avoid two collisions. The *primary collision* occurs when a

*Correspondence to: Peng-Jun Wan, Department of Computer Science, Illinois Institute of Technology, Chicago, IL 60616, U.S.A.
†E-mail: wan@cs.iit.edu

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node simultaneously transmits and receives signals over the same channel. The *secondary collision* occurs when a node simultaneously receives more than one-signals over the same channel. Thus, to prevent the primary collision, two nodes can be assigned the same channel if and only if neither of them is within the transmission range of the other. Similarly, to prevent the secondary collision, two nodes can be assigned the same channel if and only if no other node is located in the intersection of their transmission ranges.

The *conflict-free channel assignment problem* seeks an assignment of the fewest channels to a given set of radio nodes with specified transmission ranges without any primary collision or secondary collision. It is a classic and fundamental problem in wireless *ad hoc* networks [1–7]. It is NP-hard even when all nodes are located in a plane and have the same transmission radii [8]. Therefore, only polynomial-time approximation algorithms can be expected. The performance of a polynomial-time approximation algorithm is measured by its approximation ratio, which is the supreme, over all instances, of the ratio of the number of channels output by this algorithm to the minimum number of channels.

The conflict-free channel assignment problem is equivalent to the minimum vertex coloring of a special class of geometric graphs. Let V be the set of given radio nodes, and r_v be the specified transmission radius of node v for each $v \in V$. For any pair of nodes u and v, we use $|uv|$ to denote their Euclidean distance. Then a geometric graph over V can be obtained by creating an edge between each pair of nodes (u, v) satisfying that either $|uv| \leq \max\{r_u, r_v\}$ or there is a node $w \in V \setminus \{u, v\}$ such that $|uw| \leq r_u$ and $|vw| \leq r_v$. The resulting graph is referred to as the *interference graph*. Then any proper vertex coloring of the interference graph gives rise to a valid channel assignment for the node scheduling of V, and vice versa. When all nodes in V have the same transmission radii, then the interference graph is the square[‡] of the unit-disk graph [9] over V in which there is an edge between a pair of nodes if and only if they are in the transmission of each other. In this case, the conflict-free channel assignment problem happens to be the same as the minimum distance-2 vertex coloring[§] [10] of unit-disk graphs. However, when the nodes in V have disparate transmission radii, the interference graph may be not the square of any graph. Indeed, for the instance V shown in Figure 1, its interference graph is a chain, which is not a square of any graph. Therefore, the conflict-free channel assignment problem is in general different from the distance-2 vertex coloring.

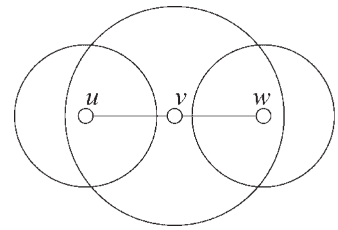


Fig. 1. The interference graph may be not the square of any graph.

First-fit coloring is a well-known greedy method for vertex coloring: Given a vertex ordering, assign the least possible color to the vertices sequentially. The number of colors can be bound in terms of the *inductivity* of the vertex ordering, which is the least integer q such that each vertex is adjacent to at most q prior vertices. Obviously, the vertex coloring induced by a vertex ordering of inductivity q uses at most $q + 1$ colors. This suggests the use of a vertex ordering of small inductivity to induce a vertex coloring. The following vertex orderings of the interference graph will be used in this paper:

1. Smallest-last ordering: This vertex ordering has the least inductivity and can be found in polynomial time using a greedy algorithm given by Matula and Beck [11].
2. Radius-decreasing ordering: The vertices are sorted in the decreasing order of their transmission radii.
3. Distance-increasing ordering: The vertices are sorted in the increasing order of their Euclidean distances from an arbitrary fixed reference point.
4. Lexicographic ordering: The vertices are sorted in the lexicographic order of their coordinates.

Corresponding to the above four vertex orderings, there are four FIRST-FIT heuristics: FIRST-FIT in smallest-last ordering, FIRST-FIT in radius-decreasing ordering, FIRST-FIT in distance-increasing ordering, and FIRST-FIT in lexicographic ordering. Sen and Malesinska [6] made the first and also the only attempt to prove an upper bound of 14 on the approximation ratio of FIRST-FIT in smallest-last ordering in the

‡The square of a graph H is the graph H^2 obtained by creating an edge between each pair of vertices of H whose graph distance in H is at most two.

§A distance-2 vertex coloring of a graph H is a proper vertex coloring of H^2, the square graph of H.

interference graph of a set of radio nodes lying in a plane. Unfortunately, their proof is quite erroneous and does not allow simple fix. In this paper, we assume that all radio nodes lie in a plane as in all prior works. We provide rigorous analyses of the approximation ratios of the four FIRST-FIT heuristics:

- The approximation ratios of both FIRST-FIT in smallest-last ordering and FIRST-FIT in radius-decreasing ordering are at most 13.
- If the ratio of the largest transmission radius to the least transmission radius is no more than $1/2\sin\left(\frac{360^\circ}{13}\right) \approx 1.076$, a better upper bound of 12 is derived on the approximation ratios of FIRST-FIT in smallest-last ordering and FIRST-FIT in radius-decreasing ordering. This result is useful when all nodes have the same nominal transmission radii but may allow small drift.
- When all nodes have equal transmission radii, we obtain an upper bound of 7 on the approximation ratios of FIRST-FIT in smallest-last ordering, FIRST-FIT in distance-increasing ordering, and FIRST-FIT in lexicographic ordering.

For nodes with equal transmission radii, we propose an additional spatial divide-and-conquer heuristic called *TILE coloring*. TILE coloring has an approximation ratio of 12, but is extremely simple in implementation and especially suitable for distributed implementation and dynamic implementation in a mobile environment.

A problem closely related to the conflict-free channel assignment problem is the maximum-independent set problem in the interference graphs. An independent set of nodes in the interference graph can share one common channel without causing any primary collision or secondary collision. We modify all the heuristics, except FIRST-FIT in smallest-last ordering, for the conflict-free channel assignment problem to heuristics for the maximum-independent set problem with the same approximation ratios.

We introduce some symbols and notations that will be used throughout this paper. The nodes in V are said to have *quasi-uniform* transmission radii if the ratio of $\max_{v\in V} r_v$ to $\min_{v\in V} r_v$ is at most $1/2\sin\left(\frac{360^\circ}{13}\right)$, and have *uniform* transmission radii is all r_v's are equal. The interference graph of V is denoted by G. We use $\chi(G), \omega(G), \alpha(G), \Delta(G)$ to denote its chromatic number, clique number, independence number, and maximum degree respectively of G. The inductivity of the smallest-last ordering is also called the *inductivity* of G and is denoted by $\delta^*(G)$.

The remaining of this paper is arranged as follows. In Section 2, we prove two geometrical properties of two intersecting circles, which are invariant to the distance between their centers. These two properties will be used in the analysis in the next section. In Section 3, we explore topological properties of the neighborhood of the node with the smallest transmission radius in the interference graph. Based on these topological properties, in Section 4 we derive upper bounds on the approximation ratios of all FIRST-FIT greedy algorithms. In Section 5, we present the algorithms TILE coloring for nodes with uniform transmission radii and analyze its approximation ratios. In Section 6, we modify all the heuristics, except FIRST-FIT in smallest-last ordering, for the conflict-free channel assignment problem to heuristics for the maximum independent set problem with the same approximation ratios. Finally, we conclude the paper in Section 7.

2. Two Invariant Geometric Properties

In this section, we present two elementary geometrical properties of two intersecting circles, which are invariant to the distance between their centers. We first present the 'equilateral triangle property.'

Lemma 1 [Equilateral Triangle Property]. *Consider two unit circles C_1 and C_2 centered at u_1 and u_2, respectively with $1 \le |u_1u_2| \le 2$ (see Figure 2). Let v_1 and v_2 be their two intersection points. Let w_1 and w_2 be the two intersection points of C_2 and the line through u_1 that has a 30°-slope from the line u_1u_2 and hits the segment u_2v_1. Then both $\triangle u_2v_2w_1$ and $\triangle u_2v_1w_2$ are equilateral.*

Proof. Consider the two isosceles triangles $\triangle u_2v_1v_2$ and $\triangle u_2w_1w_2$. Their sides are one and their heights are $|u_1u_2|/2$, and as such they are identical. Thus,

$$\begin{aligned} v_2\widehat{u_2}w_1 &= u_1\widehat{u_2}v_2 + u_1\widehat{u_2}w_1 = u_1\widehat{u_2}v_2 + u_2\widehat{w_1}w_2 - 30^\circ \\ &= u_1\widehat{u_2}v_2 + u_2\widehat{v_2}v_1 - 30^\circ = 90^\circ - 30^\circ = 60^\circ \end{aligned}$$

Since $v_1\widehat{u_2}v_2 = w_1\widehat{u_2}w_2$, we have $v_2\widehat{u_2}w_1 = v_1\widehat{u_2}w_2$. Therefore, both $\triangle u_2v_2w_1$ and $\triangle u_2v_1w_2$ are equilateral. ■

Now we present the second invariant property.

Lemma 2. *Let C_1 be a circle of radius 1 centered at u_1, and C_2 be a circle of radius $r \ge 1$ centered at u_2 with $r \le |u_1u_2| \le r+1$ (see Figure 3). Let v be an intersection point of C_1 and C_2, and w be any point in*

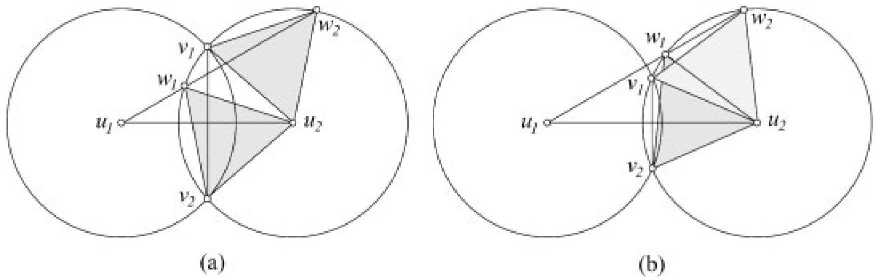


Fig. 2. Equilateral triangle property.

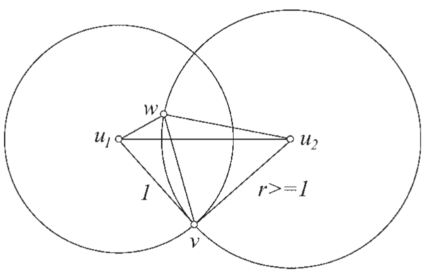


Fig. 3. $|vw| \leq 1$ if $\widehat{u_2 u_1 w} \leq \arcsin \frac{1}{2r}$.

C_2 that lies inside the disk bounded by C_1. Then $|vw| \leq 1$ if $\widehat{u_2 u_1 w} \leq \arcsin(1/2r)$.

Proof. Since $|vw|$ increases as w moves away from v along C_2, we only need to consider the point w that lies in the opposite side of u_1u_2 from v with $\widehat{u_2 u_1 w} = \arcsin(1/2r)$. Then in $\triangle u_1 u_2 w$, $\widehat{u_1 u_2 w} \leq \widehat{u_2 u_1 w} \leq 30^\circ$. So $\widehat{u_1 w u_2}$ is obtuse. Let d denote $|u_1u_2|$. By applying the law of sines to $\triangle u_1 u_2 w$, $\widehat{u_1 w u_2} = 180^\circ - \arcsin(d/2r^2)$. Thus,

$$\widehat{u_1 u_2 w} = \arcsin \frac{d}{2r^2} - \arcsin \frac{1}{2r}$$

By applying the law of cosines to $\triangle u_1 u_2 v$,

$$\widehat{u_1 u_2 v} = \arccos \frac{d^2 + r^2 - 1}{2rd}$$

Therefore,

$$\widehat{u_1 u_2 w} = \arccos \frac{d^2 + r^2 - 1}{2rd} + \arcsin \frac{d}{2r^2} - \arcsin \frac{1}{2r}$$

We claim that $\widehat{u_1 u_2 w}$ is a decreasing function of d for $d \geq r$. Indeed, the derivative of $\widehat{u_1 u_2 w}$ with respect to d is

$$\begin{aligned} &\frac{1/2r^2}{\sqrt{1 - \left(\frac{d}{2r^2}\right)^2}} - \frac{(1/2r) - (r^2-1)/2rd^2}{\sqrt{1 - \left(\frac{d}{2r} + \frac{r^2-1}{2r}\frac{1}{d}\right)^2}} \\ &= \frac{1}{\sqrt{4r^4 - d^2}} - \frac{1 - (r^2-1)/d^2}{\sqrt{4r^2 - \left(d + \frac{r^2-1}{d}\right)^2}} \end{aligned}$$

Since $d \geq r$,

$$\begin{aligned} &\left[4r^2 - \left(d + \frac{r^2-1}{d}\right)^2\right] - \left(4r^4 - d^2\right)\left(1 - \frac{r^2-1}{d^2}\right)^2 \\ &= 4r^2 + 4r^4\left(1 - \frac{r^2-1}{d^2}\right)^2 - 4\left(r^2 - 1\right) \\ &= 4 - 4r^4\left(1 - \frac{r^2-1}{d^2}\right)^2 \\ &\leq 4 - 4r^4\left(1 - \frac{r^2-1}{r^2}\right)^2 = 0 \end{aligned}$$

Thus, the derivative of $\widehat{u_1 u_2 w}$ with respect to d is non-positive for $d \geq r$, and as such our claim is true. Therefore, $\widehat{u_1 u_2 w}$ achieves its maximum at $d = r$, so is $|vw|$. However, when $d = r$ the point w coincides with u_1 and thus the maximum of $|vw|$ is 1. ■

3. Neighborhood of The Node with The Smallest Transmission Radius

Two nodes are said to be adjacent if they are neighbors in the interference graph. We distinguish two types of neighbors. Let u be any node. A neighbor v of u is said to be a *primary neighbor* of u if $|uv| \leq \max\{r_u, r_v\}$, and a *secondary neighbor* of u otherwise. For simplicity, we use B_v to denote the disk centered at v with radius r_v, and C_v to denote the boundary circle of B_v.

In the remaining of this section, we fix u to be the node with minimum transmission radius. By proper scaling, we assume that $r_u = 1$. We study the sufficient conditions for two neighbors of u to be adjacent.

The next lemma shows that any two primary neighbors of u are adjacent.

Lemma 3. *Suppose that v and w are two primary neighbors of u. Then v and w are adjacent. In addition, if $\widehat{vuw} \le 60°$ and $|uv| \le \max\{|uw|, 1\}$, then $v \in B_w$.*

Proof. If v and w are not primary neighbors to each other, then they must be secondary neighbors to each other as $u \in B_v \cap B_w$. If $\widehat{vuw} \le 60°$ and $|uv| \le \max\{|uw|, 1\}$, then in $\triangle uvw$,

$$|vw| \le \max\{|uv|, |uw|\} \le \max\{|uw|, 1\} \le r_w$$

Thus, $v \in B_w$. ■

The lemma below gives a sufficient condition for two secondary neighbors of u to be adjacent.

Lemma 4. *Suppose that v and w are two secondary neighbors of u. Then v and w are adjacent if $\widehat{vuw} \le 2 \arcsin(1/4) \approx 28.955°$.*

Proof. Let x and y be the points where C_u meets uv and uw, respectively. We consider two complementary cases. In the first case, either $x \in B_w$ or $y \in B_v$. In the second case, both $y \notin B_v$ and $x \notin B_w$.

Case 1. Either $x \in B_w$ or $y \in B_v$. We prove a *stronger* result that v and w are adjacent if $\widehat{vuw} \le 30°$. Assume $\widehat{vuw} \le 30°$. By symmetry we assume $y \in B_v$. Note that the lemma follows if $w \in B_v$, so we also assume that $w \notin B_v$. We further consider two complementary subcases. In the first subcase, $|vy| \ge 1$ (see Figure 4(a)). In the second subcase, $|vy| < 1$ (see Figure 4(b)).

Subcase 1.1. $|vy| \ge 1$. We claim that $v \in B_w$. Indeed in $\triangle uvy$, $\widehat{uvy} \le \widehat{vuy} \le 30°$. Thus, $\widehat{vyw} = \widehat{vuy} + \widehat{uvy} \le 60°$. In $\triangle vwy$, since $|vy| \le r_v < |vw|$, $\widehat{vwy} < \widehat{vyw} < 60°$ and as such $\widehat{wvy} > 60° > \widehat{vyw}$. So $|wy| > |wv|$. As $y \in B_v$, our claim follows.

Subcase 1.2. $|vy| < 1$. We claim that either $v \in B_w$ or $B_u \cap B_w \subseteq B_v$. Let z be the intersection point of the two circles C_u and C_v that lies in the same side of uv as w. We show that $|wz| > |wv|$. Since $|wz|$ decreases and $|wv|$ increases while rotating uw away from uv, we can restrict $\widehat{vuw}$ to 30°. Since $|wz|$ decreases and $|wv|$ remains unchanged while shrinking the disk B_v, we can further restrict r_v to 1 (see Figure 4(b)). Let w' be the intersection point of wy and C_v. Then $|w'z| = |w'v|$ by Lemma 1. Consider $\triangle uw'z$ and $\triangle uw'v$. As $|uz| < |uv|$, $\widehat{uw'z} < \widehat{uw'v}$. Now consider $\triangle ww'z$ and $\triangle ww'v$. As $\widehat{ww'z} > \widehat{ww'v}$, $|wz| > |vv|$. So if $r_w \ge |wz|$, then $v \in B_w$; if $r_w < |wz|$, then $B_u \cap B_w \subseteq B_v$. Therefore, our claim is true.

Case 2. Both $y \notin B_v$ and $x \notin B_w$. By symmetry we assume that $|uv| \le |uw|$. We claim that $v \in B_w$. Let v' be the point in the ray uv satisfying that $|v'y| = 1$, and v'' be the point in the ray uv satisfying that $|uv''| = |uw|$ (see Figure 5). Then $|uv'| < |uv''|$, and v lies on $v'v''$. So it is sufficient to show that both $v' \in B_w$ and $v'' \in B_w$. The former follows if $|v'w| \le 1$ and follows from the same argument as in *Subcase 1.1.* if $|v'w| > 1$. The latter is true as

$$\begin{aligned}|v''w| &= 2|uw| \sin \frac{\widehat{vuw}}{2} \\ &= 2(1 + |wy|) \sin \frac{\widehat{vuw}}{2} \le \frac{1 + |wy|}{2} \\ &\le \max\{1, |wy|\} \le r_w\end{aligned}$$

So our claim is true. ■

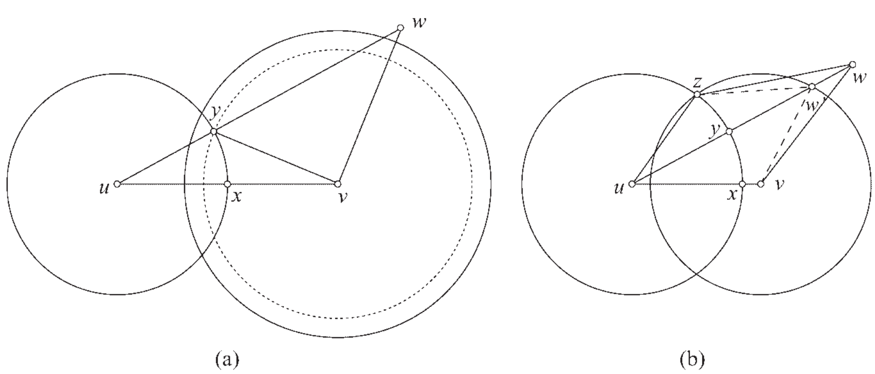


Fig. 4. *Case 1* in the proof of Lemma 4. (a) $|vy| \ge 1$; (b) $|vy| < 1$.

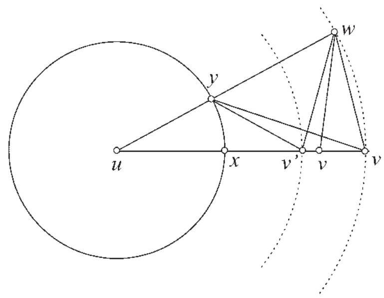


Fig. 5. *Case 2* in the proof of Lemma 4.

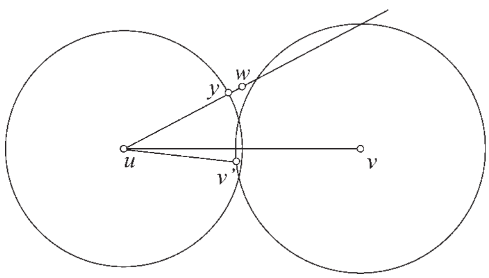


Fig. 6. *Case 1* in the proof of Lemma 5.

The next lemma gives two sufficient conditions for a primary neighbor of u and a secondary neighbors of u to be adjacent.

Lemma 5. *Suppose that v is a secondary neighbor of u, and w is a primary neighbor of u. Then v and w are adjacent if either $\widehat{vuw} \leq 30°$ and $|uw| \geq 1$ or $\widehat{vuw} \leq \arcsin(1/2r_v)$.*

Proof. The lemma follows if $w \in B_v$, so we assume that $w \notin B_v$. Let y be the intersection point of C_u and the ray uw. We consider two complementary cases. In the first case, $y \notin B_v$. In the second case, $y \in B_v$.

Case 1: $y \notin B_v$ (see Figure 6). We claim that $B_u \cap B_v \subseteq B_w$ if $\widehat{vuw} \leq 30°$. Assume $\widehat{vuw} \leq 30°$. Let v' be any point in $B_u \cap B_v$. Then $\widehat{wuv'} \leq \widehat{vuw} + \widehat{vuv'} \leq 2\widehat{vuw} \leq 60°$. As $|uv'| \leq 1$, $v' \in B_w$byLemma 3. Thus, $B_u \cap B_v \subseteq B_w$.

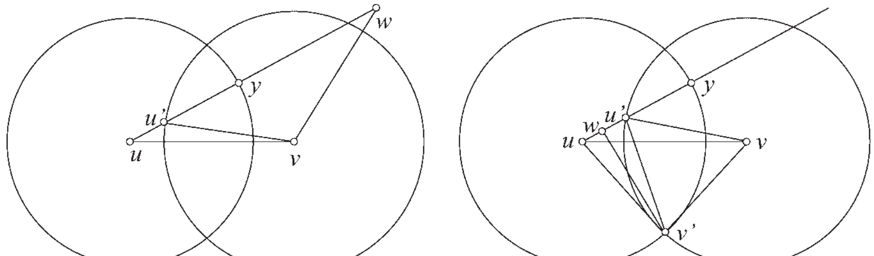
Case 2. $y \in B_v$. Let u' be the intersection point of C_v and uy. We further consider two subcases depending on whether w lies either on uu' or not. In the first subcase, $|uw| \geq 1$ (see Figure 7(a)). In the second subcase, $|uw| < 1$ (see Figure 7(b)).

Subcase 2.1. $|uw| \geq 1$. Then w does not lie on uu'. We claim that $v \in B_w$ if $\widehat{vuw} \leq 30°$. Assume $\widehat{vuw} \leq 30°$. Then in $\triangle u'uv$, since the edge uu' is the shortest one, $\widehat{uvu'} \leq \widehat{uvu'} \leq 30°$. Thus, $\widehat{vu'w} = \widehat{uvu'} + \widehat{u'uv} \leq 60°$. In $\triangle u'vw$, as $|u'v| = r_v < |vw|$, $\widehat{u'wv} < \widehat{vu'w} \leq 60°$. So in $\triangle uvw$, $\widehat{uvw}$ is obtuse. This implies that $|uw| > |vw|$. As $r_w \geq |uw|$, $v \in B_w$.

Subcase 2.2. $|uw| < 1$. Then w lies on uu'. We claim that $B_u \cap B_v \subseteq B_w$ if $\widehat{vuw} \leq \arcsin(1/2r_v)$. Assume $\widehat{vuw} \leq \arcsin(1/2r_v)$. We will prove a *stronger* result that every point in $B_u \cap B_v$ is at a distance of at most 1 from w. Since w lies on uu', it is sufficient to show that every point in $B_u \cap B_v$ is at a distance of at most 1 from u'. Let v' be the point in $B_u \cap B_v$ such that $|u'v'|$ is the largest. Obviously, v' is in the boundary of $B_u \cap B_v$. By applying the law of cosines to $\triangle u'uv'$ and $\triangle u'vv'$, we further observe that v' must be the intersection point of C_u and C_v that lies in the different side of uv from u'. Since $\widehat{vuw} \leq \arcsin(1/2r_v)$, $|u'v'| \leq 1$ by Lemma 2. Thus, every point in $B_u \cap B_v$ is at a distance of at most 1 from u'. ■

Now we partition the plane into 14 regions as shown in Figure 8(a). One region is a unit disk centered at u. Each of the other 13 infinite regions is surrounded by the boundary of this unit disk and two rays emanated from u and separated by $(360°/13) \approx 27.7°$. The next lemma shows that all neighbors of u in each region form a clique.

Lemma 6. *All neighbors of u lying in each of the 14 regions in the partition shown in Figure 8(a) form a clique. Consequently, u has at most 13 independent (i.e., pairwise nonadjacent) neighbors.*



(a) (b)

Fig. 7. *Case 2* in the proof of Lemma 5. (a) $|uw| \geq 1$; (b) $|uw| < 1$.

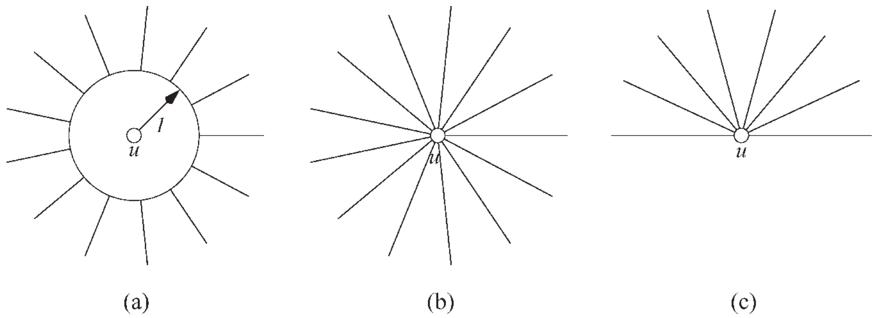


Fig. 8. Three partitions of the neighborhood of u.

Proof. It is sufficient to show that any two neighbors, v and w, of u lying in the same region are adjacent. If v and w lie in the unit disk, then both of them are primary neighbors of u and hence are adjacent by Lemma 3. So we assume that they lie in one of 13 infinite regions. If they are both primary neighbors of u, then they are adjacent by Lemma 3. If they are both secondary neighbors of u, then they are also adjacent by Lemma 4 since $\widehat{vuw} \leq (360^\circ/13) < 2\arcsin(1/4)$. If one of them is a primary neighbor of u and the other is a secondary neighbor of u, then they are adjacent by Lemma 5 since $\widehat{vuw} \leq (360^\circ/13) < 30^\circ$, $|uv| \geq 1$, and $|uw| \geq 1$. ■

If the transmission radii of all neighbors of u are at most $1/2\sin(360^\circ/13) \approx 1.076$, we adopt the partition shown in Figure 8(b). Each of the 13 regions is surrounded by two rays emanated from u and separated by $(360^\circ/13) \approx 27.7^\circ$.

Lemma 7. *Suppose that the transmission radii of all neighbors of u are at most* $1/2\sin(360^\circ/13)$. *Then all neighbors of u lying in each of the 13 regions in the partition shown in Figure 8(b) form a clique. Consequently, u has at most 12 independent neighbors.*

Proof. It is sufficient to show that any two neighbors, v and w, of u lying in the same region are adjacent. If v and w are both primary neighbors of u or are both secondary neighbors of u, then they are also adjacent following the same argument as in the proof of Lemma 6. If one of them is a primary neighbor of u and the other is a secondary neighbor of u, then they are adjacent by Lemma 5 since $\widehat{vuw} \leq (360^\circ/13) \leq \arcsin 1/2r_v$ and similarly $\widehat{vuw} \leq \arcsin(1/2r_w)$. ■

Finally, if all neighbors of u lie in a half-plane and have transmission radii of at most $1/2\sin(180^\circ/7) \approx 1.152$, we adopt the partition shown in Figure 8(c) of the half-plane containing all neighbors of u. Each of the seven regions is surrounded by two rays emanated from u and separated by $(180^\circ/7) \approx 25.7^\circ$. Following the similar argument in the proof of Lemma 7, we can prove the following lemma.

Lemma 8. *Suppose that all neighbors of u lie in a half-plane and have transmission radii of at most* $(1/2\sin(180^\circ/7)$. *Then all neighbors of u lying in each of the seven regions in the partition shown in Figure 8(c) of the half-plane containing all neighbors of u form a clique. Consequently, u has at most seven independent neighbors.*

4. FIRST-FIT Coloring

Since FIRST-FIT in a vertex ordering of inductivity q uses at most $q+1$ colors, our analyses of the FIRST-FIT colorings will be based on the upper bounds on the inductivities of the corresponding vertex orderings. Since the smallest-last ordering has the smallest inductivity, an upper bound on the inductivity of any other vertex ordering applies to the smallest-last ordering as well.

We begin with upper-bounding the inductivity of the radius-decreasing ordering.

Lemma 9. *The inductivity of any radius-decreasing ordering is at most* $\min\{14\omega(G) - 2, 13\chi(G) - 13\}$. *If all nodes have quasi-uniform transmission radii, then the inductivity of any radius-decreasing ordering is at most* $\min\{13\omega(G) - 14, 12\chi(G) - 12\}$.

Proof. Consider an arbitrary radius-decreasing ordering and let q be its inductivity. Let u be a node with q prior neighbors in this ordering. Note that the transmission radii of these q prior neighbors of u are no less than that of u. By proper scaling, we can assume that the transmission radii of u is one. Then the transmission radii of these q prior neighbors of u are at least one.

We first show that $q \leq 14\omega(G) - 2$. If all these q prior neighbors of u lie in the unit disk centered at u, then they together with u form a clique by Lemma 6 and thus $q \leq \omega(G) - 1 < 14\omega(G) - 2$. So we assume that some of these q prior neighbors of u lie outside the unit disk centered at u. We partition the plane as shown in Figure 8(a) such that at least one of these q prior neighbors lie in the boundary of one of those 13 infinite regions. Thus, at most $13\omega(G) - 1$ prior neighbors of u lie outside the unit disk centered at u. Also at most $\omega(G) - 1$ prior neighbors of u lie in the unit disk centered at u. Therefore, $q \leq (13\omega(G) - 1) + (\omega(G) - 1) = 14\omega(G) - 2$.

Now we show that $q \leq 13\chi(G) - 13$. By Lemma 6, at most 13 prior neighbors of u are independent can get the same color in any proper coloring. Therefore, the prior neighbors of u requires at least $\lceil \frac{q}{13} \rceil$ colors. Since u must be colored differently than its neighbors, u and all its q prior neighbors requires at least $1 + \lceil \frac{q}{13} \rceil$ colors. This implies that $\chi(G) \geq 1 + \lceil \frac{q}{13} \rceil$. Therefore, $q \leq 13\chi(G) - 13$.

If all nodes have quasi-uniform transmission radii, then following from a similar argument and Lemma 7, we can show that $q \leq \min\{13\omega(G) - 14, 12\chi(G) - 12\}$. The detail is omitted. ■

From the above lemma, we have $\delta^*(G) \leq \min\{14\omega(G) - 2, 13\chi(G) - 13\}$. If all nodes have quasi-uniform transmission radii, then $\delta^*(G) \leq \min\{13\omega(G) - 14, 12\chi(G) - 12\}$. Thus, we have the following theorem.

Theorem 10. *Both FIRST-FIT in smallest-last ordering and FIRST-FIT in radius-decreasing ordering use at most* $\min\{14\omega(G) - 1, 13\chi(G) - 12\}$ *colors, and hence have approximation ratios of at most 13. If all nodes have quasi-uniform transmission radii, then both of them use at most* $\min\{13\omega(G) - 13, 12\chi(G) - 11\}$ *colors, and hence have approximation ratios of at most 12.*

As a corollary of the above theorem, $\omega(G) \leq \chi(G) \leq 14\omega(G) - 1$. If all nodes have quasi-uniform transmission radii, $\omega(G) \leq \chi(G) \leq 13\omega(G) - 13$. We also remark that when all nodes have the uniform transmission radii, then every vertex ordering is a radius-decreasing ordering. In this case, FIRST-FIT in an *arbitrary* vertex ordering still has an approximation ratio of at most 12. This also follows from the fact if all nodes have the uniform transmission radii, then $\Delta(G) \leq \min\{13\omega(G) - 14, 12\chi(G) - 12\}$. The proof of this fact is similar to that of Lemma 9 and is omitted here.

Next, we upper bound the inductivities of distance-decreasing ordering and lexicographic ordering when all nodes have uniform transmission radii.

Lemma 11. *Suppose that all nodes have uniform transmission radii. Then the inductivity of any distance-increasing ordering or any lexicographic ordering is at most* $7\omega(G) - 7$.

Proof. A key property of both distance-increasing ordering and any lexicographic ordering is that for each node u, all its prior neighbors all lie in a half-plane with u at the boundary. Because of this property, Lemma 8 can be applied. The theorem can then be proven by following the similar but simpler argument to the proof of Lemma 9. The detail is omitted. ■

From the above lemma, if all nodes have uniform transmission radii, then $\delta^*(G) \leq 7\omega(G) - 7$. Thus, we have the following theorem.

Theorem 12. *Suppose that all nodes have uniform transmission radii. Then FIRST-FIT in smallest-last ordering, FIRST-FIT in distance-increasing ordering, and FIRST-FIT in lexicographic ordering all use at most* $7\omega(G) - 6$ *colors, and hence have approximation ratios of at most 7.*

As a corollary of the above theorem, if all nodes have uniform transmission radii, then $\omega(G) \leq \chi(G) \leq 7\omega(G) - 6$.

5. TILE Coloring

Throughout of this section, we assume that all nodes have uniform transmission radii equal to one. Under this assumption, the interference graph is the square of the unit-disk graph over all the nodes. We propose a spatial divide-and-conquer heuristic referred to as *TILE coloring*.

In this heuristic, we tile the plane into regular hexagons of side equal to $1/2$ (see Figure 9). Each hexagon, or cell, is considered to be left-closed and right open, with the top-most point included and the bottom-most point excluded (see Figure 10). Cells are further grouped into clusters of size 12 according to the pattern as shown in Figure 9. We then label the 12 hexagons in a cluster with the numbers 1 through 12 in an arbitrary pattern, and repeat the same labeling for all clusters. For simplicity, each color is represented by a duple (c_1, c_2) where $1 \leq c_1 \leq 12$ and c_2 is a positive integer. For each nonempty hexagon labeled with i, all nodes inside it are colored as follows. We first sort the nodes in this hexagon in an arbitrary order

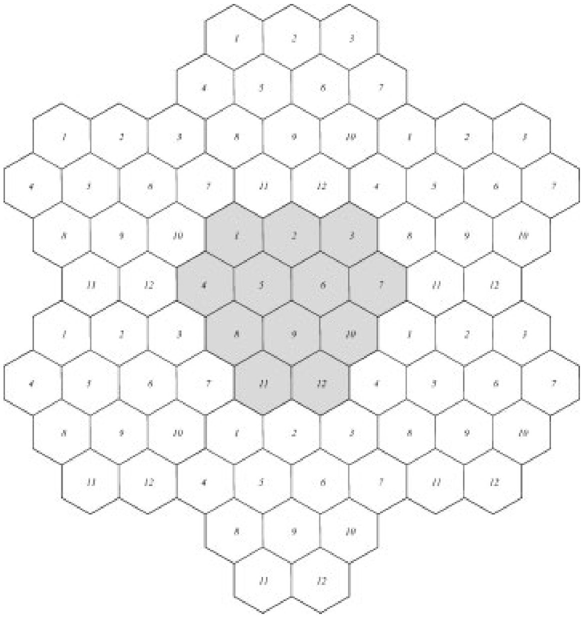


Fig. 9. Tiling of the plane into hexagons with 12 hexagons per cluster.

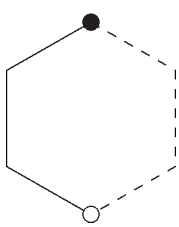


Fig. 10. Half-closed half-open hexagon.

(e.g., in the increasing order of the node ID). Then the jth node in this order receive the color (i,j).

Clearly, TILE coloring produces a valid coloring, since the distance between any two (half-closed and half-open) hexagons with the same label is greater than 2. As all nodes within a hexagon form a clique (more precisely, a clique of the unit-disk graph), the total number of colors is at most $12\omega(G)$. So we have the following performance result on TILE coloring.

Theorem 13. *TILE coloring uses at most $12\omega(G)$ colors, and hence has an approximation ratio of at most 12.*

6. Heuristics for Maximum Independent Set

In this section, we modify all the heuristics, except FIRST-FIT in smallest-last ordering, for the conflict-free channel assignment problem to heuristics for the maximum independent set problem with the same approximation ratios. In general, the first-fit heuristic in a specified vertex ordering starts with an empty independent set, and sequentially adds a vertex which is not adjacent to any vertex in the current independent set. The following vertex orderings of the interference graph will be adopted to produce the independent sets:

1. Radius-increasing ordering: the vertices are sorted in the increasing order of their transmission radii.
2. Distance-decreasing ordering: the vertices are sorted in the decreasing order of their Euclidean distances from an arbitrary fixed reference point.
3. Lexicographic ordering.

Theorem 14. *FIRST-FIT in radius-increasing ordering has an approximation ratio of at most 13 in general, and an approximation ratio of at most 12 if all nodes have quasi-uniform transmission radii. If all nodes have uniform transmission radii, both FIRST-FIT in distance-decreasing ordering and FIRST-FIT in lexicographic ordering have approximation ratios of at most 7.*

Proof. Let $u_1, u_2, \ldots, u_m$ denote the sequence of nodes added to the independent set produced by a FIRST-FIT heuristic in a given vertex ordering. Obviously, they form a maximal independent set. For $1 \leq i \leq m$, let U_i denote the set of its rear-neighbors in the given ordering plus u_i itself. Then,

$$V = U_1 \cup U_2 \cup \cdots \cup U_m$$

Assume the given vertex ordering is the radius increasing ordering. Then u_i is the node in U_i with the smallest transmission radius. By Lemma 6, each U_i contains at most 13 independent nodes, and thus the independence number is at most $13m$. This implies the approximation ratio of FIRST-FIT in radius-increasing ordering is at most 13. If all nodes have quasi-uniform transmission radii, then by Lemma 7 each U_i contains at most 12 independent nodes, and thus the independence number is at most $12m$. In this case, the approximation ratio of FIRST-FIT in radius-increasing ordering is at most 12.

Now assume the given vertex ordering is either the distance-decreasing ordering or the lexicographic ordering, and all nodes have uniform transmission radii. Then all nodes in U_i lie in a half-plane with u at the boundary. By Lemma 8, each U_i contains at most seven independent nodes, and thus the independence number is at most $7m$. This implies the approximation ratio of either FIRST-FIT in distance-decreasing ordering or FIRST-FIT in lexicographic ordering is at most 7. ■

Next, we describe how to modify TILE coloring to a heuristic, referred to as *TILE IS*, for maximum independent set. We tile the plane into hexagons and label the hexagons as in TILE coloring. Let i be the label such that the number of non-empty hexagons with label i is the largest one. We choose an arbitrary node from each non-empty hexagons with label i. These chosen nodes form an independent set, which is the output of TILE IS. Note that the size of this independent set is at least one-twelfth of the number of non-empty hexagons. On the other hand, any independent set contains at most one node from each non-empty hexagon. Thus, the independence number is no more than the number of non-empty hexagons. This implies that the approximation ratio of TILE IS is at least 12. So we have the following theorem.

Theorem 15. *TILE IS has an approximation ratio of at most 12.*

7. Conclusion

Conflict-free channels assignment is a classic and fundamental problem in wireless *ad hoc* networks. It is NP-hard even when all nodes are located in a plane and have equal transmission radii. We observe that a prior analysis by Sen and Malesinska [6] of the approximation ratio of FIRST-FIT in smallest-last ordering is erroneous. In this paper, we provide a rigorous and tighter analysis of this algorithm and other greedy FIRST-FIT algorithms, FIRST-FIT in radius-decreasing ordering, FIRST-FIT in distance increasing ordering, and FIRST-FIT in lexicographic ordering. In addition, for nodes with equal transmission radii, we present two spatial divide-and-conquer algorithms, TILE coloring and STRIP coloring and analyze their approximation ratios. We also obtain relations among the three parameters of the interference graph: inductivity, clique number, and the chromatic number. In addition, we modify all the heuristics, except FIRST-FIT in smallest-last ordering, for the conflict-free channel assignment problem to heuristics for the maximum independent set problem with the same approximation ratios.

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Authors' Biographies

Dr. Peng-Jun Wan received his Ph.D. (1997) from the University of Minnesota, M.S. degree (1993) from the Chinese Academy of Science, and B.S. degree (1990) from Tsinghua University. He is currently an associate professor in Computer Science at Illinois Institute of Technology. His research interests include wireless networks and optical networks.

Mr. Chih-Wei Yi received his M.S. and B.S. degree in mathematics (1991) and M.S. degree in Computer Science (1993) from National Taiwan University. He is currently a Ph.D. candidate in Department of Computer Science at the Illinois Institute of Technology. His dissertational research focuses on wireless *ad hoc* networks. He is expected to graduate in 2005.

Dr. Xiaohua Jia received his B.Sc. (1984) and M.Eng. (1987) from the University of Science and Technology of China, and obtained his D.Sc. (1991) in information science from the University of Tokyo, Japan. He is currently a professor in Department of Computer Science at City University of Hong Kong, adjunct with School of Computing, Wuhan University, China. Professor Jia's research interests include distributed systems, computer networks, WDM optical networks, Internet technologies, and mobile computing.

Dr. Dongsoo S. Kim received the B.E. degree (1987) from Korea University, M.S. degree (1994) from the Unversity of Texas at Dallas, Ph.D. (1998) from the University of Minnesota. He is currently an assistant professor of the Department of Electrical and Computer Engineering at Indiana University Purdue University at Indianapolis, Indiana. His research interests include wireless networks and optical networks.

