# cs330 - Discrete Structures <br> Spring 1998 

Final Exam
closed books, closed notes

Starts: 10:30 am
Ends: 12:30 pm
Name: $\qquad$ (please print)

ID: $\qquad$

| Problem | Max points | Your mark | Comments |
| :---: | :---: | :---: | :---: |
| 1 | 10 |  |  |
| 2 | 10 |  |  |
| 3 | 10 |  |  |
| 4 | 10 |  | $10 * 1$ |
| 5 | 10 |  | $5+5$ |
| 6 | 10 |  |  |
| 7 | 10 |  | $5+5$ |
| 8 | 10 |  |  |
| 9 | 10 |  | $8 * 5$ |
| 10 | 10 |  |  |
| 11 | 40 |  |  |

What happens with this paper (mark one): Discard Mail at this address:
$\square$

1. Let $f$ be the function that associates each person on Earth his/her height (expressed as an integer number of inches). Decide whether $f$ is bijective or not.
2. Consider the set of all regular expressions over the alphabet $\mathrm{A}=\{0,1\}$. Decide whether this set is countable or not. Prove your answer (a correct guess earns you $1 / 3$ of the credit for this problem).
3. Decide whether the following argumentation is valid or not. "The Discrete Structures class is fun or is boring. If Discrete Structures is fun then students learn. Therefore, if Discrete

Structures is boring students do not learn."
4. Give an inductive definition for the set of all simple lists over the alphabet $A=\{a, b\}$ with the property that the first and the last element in each list are the same.
5. Find a regular expression for the language consisting of strings that represent numbers in scientific normal notation. For the exponent you will use a caret ( $\wedge$ ). For example, instead of $10^{2}$ you would use $10^{\wedge} 2$.
6. Determine whether the strings in the table belong to any of the languages described by the following regular expressions:

| RE | 1001 belongs to the language (T/F) | 110 belongs to the language (T/F) |
| :--- | :--- | :--- |
| $10^{*} 1^{*}$ |  |  |
| $(10)^{\star}+(1)^{\star}$ |  |  |
| $(00)^{\star} 1^{*}(01)^{\star} 1$ |  |  |
| $(00)^{*} 1^{\star}(01)^{\star} 1$ |  |  |
| $0^{*}(10+1)^{*}$ |  |  |

7. Assume a FA described by the following state transition table:

|  | Input |  |
| :---: | :---: | :---: |
| State | 0 | 1 |
| $\rightarrow * \mathrm{~s}_{0}$ | $\mathrm{~s}_{0}$ | $\mathrm{~s}_{1}$ |
| $\mathrm{~s}_{1}$ | $\mathrm{~s}_{3}$ | $\mathrm{~s}_{4}$ |
| $* \mathrm{~s}_{2}$ | $\mathrm{~s}_{2}$ | $\mathrm{~s}_{4}$ |
| $\mathrm{~s}_{3}$ | $\mathrm{~s}_{3}$ | $\mathrm{~s}_{3}$ |
| $\mathrm{~s}_{4}$ | $\mathrm{~s}_{3}$ | $\mathrm{~s}_{2}$ |

a) Draw the state transition diagram for this FA
b) Decide which of the following strings are accepted by this FA

| String | Accepted (T/F) |
| :---: | :---: |
| 00110 |  |
| 011 |  |
| 0000 |  |
| 11000 |  |
| $\varepsilon$ |  |

8. Construct a finite-state machine that takes an input string consisting of 0 's and 1 's and outputs 1 when the last two characters read are different, and 0 otherwise. Use the Mealy model.
9. $G$ is a context free grammar defined by:
$N=\{S, A\} \quad T=\{0,1\} \quad$ Start symbol: $S$
Productions:
$S \rightarrow 0 S$
$S \rightarrow 1 A$
$S \rightarrow 1$
$S \rightarrow \lambda$
$A \rightarrow 1 A$

$$
\begin{equation*}
A \rightarrow 1 \tag{6}
\end{equation*}
$$

a) what is the language of $G$ ?
$\square$
b) find a leftmost derivation of the string 00111. At each step show the production used.

10. Find the binary single-precision representation for 9.375
11. Give a definition for:
a) Set partition

b) Symmetric relation
c) Connected graph
d) Binary tree

e) The inverse of a function
$\square$
f) The grammar of a language
g) Regular Language
h) A string being accepted by a DFA

