Lecture 2

8/25

A simple expression language

"Backus-Naur Form" (BNF)

Syntax

Expressions $e ::=$ or ↘ ↑ (integer)

↑ ↑ | "s" (string)

metavar are defined by | $e_1 + e_2$ ← represent other exps

| $e_1 \hat{} e_2$ (concatenation)

| $|s|$ (length)

ex. $|$"Hello" ^ "World"$|$

Semantics

"Dynamic semantics" / "operational semantics"

- how programs evaluate

Structural operational semantics / "small step" show process of evaluation.

2 judgments

$e \mapsto e'$ "e evaluates to e' in one step" / "e steps to e'"

e val "e is a value" (done evaluating)

$$\frac{}{\overline{n}\ \text{val}}\ (V\text{-}1) \qquad \frac{}{\text{"}s\text{"}\ \text{val}}\ (V\text{-}2)$$

$$\frac{}{\overline{n_1} + \overline{n_2} \mapsto \overline{n_1+n_2}}\ (S\text{-}1) \qquad \frac{}{\text{"}s_1\text{"} \hat{} \text{"}s_2\text{"} \mapsto \text{"}s_1 s_2\text{"}}\ (S\text{-}2) \qquad \frac{}{|\text{"}s\text{"}| \mapsto \overline{|s|}}\ (S\text{-}3)$$

Ex. $|\text{"Hello"}| \mapsto 5$

$\overline{1} + \overline{2} \mapsto \overline{3}$

"Hello" ^ "World" $\mapsto$ "Hello World"

|"Hello" ^ "World"| $\mapsto$?

$$\frac{e_1 \mapsto e_1'}{e_1 + e_2 \mapsto e_1' + e_2}\ (S\text{-}4) \qquad \frac{e_2 \mapsto e_2'}{\overline{n_1} + e_2 \mapsto \overline{n_1} + e_2'}\ (S\text{-}5)$$

$$\frac{e_1 \mapsto e_1'}{e_1 \hat{} e_2 \mapsto e_1' \hat{} e_2}\ (S\text{-}6) \qquad \frac{e_2 \mapsto e_2'}{\text{"}s_1\text{"} \hat{} e_2 \mapsto \text{"}s_1\text{"} \hat{} e_2'}\ (S\text{-}7)$$

"Search" rules Left-to-right

$$\frac{e \mapsto e'}{|e| \mapsto |e'|}\ (S\text{-}8)$$

Ex. |"Hello" ^ "World"| $\mapsto$ |"Hello World"| $\mapsto$ 10

$(1+2)+(3+4) \mapsto 3+(3+4) \mapsto 3+7 \mapsto 10$

$e \mapsto^* e'$ "e evaluates to e'" (in any # of steps)

$$\frac{}{e \mapsto^* e}\ (1) \qquad \frac{e \mapsto e' \quad e' \mapsto^* e''}{e \mapsto^* e''}\ (2)$$

$e \mapsto^n e'$ "e evaluates to e' in n steps"

$$\frac{}{e \mapsto^0 e}\ (3) \qquad \frac{e \mapsto e' \quad e' \mapsto^n e''}{e \mapsto^{n+1} e''}\ (4)$$

Thm: $e \mapsto^* e'$ if and only if $e \mapsto^n e'$ for some $n \geq 0$

Pf. $\Rightarrow$ By induction on the derivation of $e \mapsto^* e'$

$\dfrac{}{e \mapsto^* e}$ then by (3), $e \mapsto^0 e$.

$\dfrac{e \mapsto e' \quad e' \mapsto^* e''}{e \mapsto^* e''}$ By IH, $e' \mapsto^n e''$. By (4), $e \mapsto^{n+1} e''$

$\Leftarrow$ By ind on the deriv of $e \mapsto^n$

$\dfrac{}{e \mapsto^0 e}$ then by (1), $e \mapsto^* e$

$\dfrac{e \mapsto e' \quad e' \mapsto^n e''}{e \mapsto^{n+1} e''}$ By IH, $e' \mapsto^* e''$. By (2), $e \mapsto^* e''$. $\square$

Ex. $|3+4| \mapsto |7| \mapsto ?$
Type error

Static semantics (type system)

Add syntax for types: Types $\tau ::= \text{int} \mid \text{string}$

Judgment: $e : \tau$ "e has type τ"

$\dfrac{}{n : \text{int}}$ (T-1) $\dfrac{}{\text{"s"} : \text{string}}$ (T-2) $\dfrac{e_1 : \text{int} \quad e_2 : \text{int}}{e_1 + e_2 : \text{int}}$ (T-3)

$\dfrac{e_1 : \text{string} \quad e_2 : \text{string}}{e_1 \hat{} e_2 : \text{string}}$ (T-4) $\dfrac{e : \text{string}}{|e| : \text{int}}$ (T-5)

Ex. $(1+2) + (3+4) : \text{int}$ $\quad |3+4| \not: \tau$ for any τ