Thm: For all $n \in \mathbb{N}$, $1+2+\ldots+2^n = 2^{n+1}-1$

Proof: By induction on n.

Base case: $n=0$

$$2^0 = 2^{0+1}-1$$
$$1 = 2-1 \checkmark$$

Inductive case:

IH: $1+2+\ldots+2^n = 2^{n+1}-1$

Show: $\underbrace{1+2+\ldots+2^n}_{=2^{n+1}-1 \text{ by IH}}+2^{n+1} = 2^{n+1+1}-1$

$$2 \cdot 2^{n+1}-1 = 2^{n+2}-1$$
$$2^{n+2}-1 = 2^{n+2}-1 \checkmark \square$$

Thm: # of edges in a tree is # of nodes -1

Proof: By structural induction on the tree

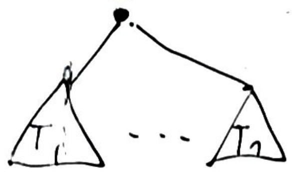
Base case: Single node

Nodes: 1

Edges: 0 ✓

Inductive case: Node w/ n children

$T =$



$$IH: \text{Edges}(T_i) = \text{Nodes}(T_i) \quad \forall i \in [1, n)$$

$$\text{Edges}(T) = n + \text{Edges}(T_1) + \dots + \text{Edges}(T_n)$$

$$\text{Nodes}(T) = 1 + \text{Nodes}(T_1) + \dots + \text{Nodes}(T_n)$$

$$\begin{aligned} \text{Edges}(T) &= \cancel{n} + \text{Nodes}(T_1) \cancel{-1} + \dots + \text{Nodes}(T_n) \cancel{-1} \quad \text{by IH} \\ &= \text{Nodes}(T_1) + \dots + \text{Nodes}(T_n) \\ &= \text{Nodes}(T) - 1 \quad \square \end{aligned}$$

$$\frac{}{\text{"" is XML}} \qquad \frac{}{\langle T\rangle \text{ is XML}} \qquad \frac{X \text{ is XML}}{\langle T\rangle X \langle /T\rangle \text{ is XML}}$$

$$\frac{X \text{ is XML} \quad Y \text{ is XML}}{XY \text{ is XML}}$$

Thm: If X is XML, then OpenAngle(X) = CloseAngle(X).

Proof: By <u>rule induction</u> on the <u>derivation</u> of X is XML

Base Case:
(Axioms)

$$\frac{}{\text{"" is XML}}$$

$OA(\text{""}) = 0 = CA(\text{""})$ ✓

$$\frac{}{\langle T\rangle \text{ is XML}}$$

$OA(\langle T\rangle) = 1 = CA(\langle T\rangle)$ ✓

Inductive cases:

$$\frac{X \text{ is XML}}{\langle T\rangle X \langle /T\rangle \text{ is XML}}$$

IH: $OA(X) = CA(X)$

$OA(\langle T\rangle X \langle /T\rangle) = 2 + OA(X)$

$CA(\langle T\rangle X \langle /T\rangle) = 2 + CA(X)$ ✓

$$\frac{X \text{ is XML} \quad Y \text{ is XML}}{XY \text{ is XML}}$$

IH: $OA(X) = CA(X)$
$OA(Y) = CA(Y)$

$OA(XY) = OA(X) + OA(Y) = CA(X) + CA(Y) = CA(XY)$ □