Basics of Parallel Programs

CS 536: Science of Programming, Fall 2021

A. *Why*

- Parallel programs are more flexible than sequential programs but their execution is more complicated.
- Parallel programs are harder to reason about because parts of a parallel program can interfere with other parts.

B. *Objectives*

At the end of this work you should be able to

- Draw evaluation graphs for parallel programs.

C. *Problems*

In general, for the problems below, if it helps you with the writing, feel free to define other symbols. ("Let $S \equiv$ *some program*," for example.)

1. What is the sequential nondeterministic program that corresponds to the program from Example 4, $[x := v \mid\mid y := v+2 \mid\mid z := v*2]$.

2. Let configuration $C_2 \equiv \langle S_2, \sigma\rangle$ *where* $S_2 \equiv [x := 1 \mid\mid x := -1]$.
   a. What is the sequential nondeterministic program that corresponds to S_1?
   b. Draw an evaluation graph for C_2.

3. Repeat Problem 2 on $C_3 \equiv \langle\ S_3, \sigma[v \mapsto 0]\ \rangle$ where $S_3 \equiv [x := v+3;\ v := v*4 \parallel v := v+2]$. Note that in the first thread, the two assignments must be done with x first, then v. Because adding 3 and adding 2 are commutative, two of the (normally-different) nodes will merge.

4. Repeat Problem 2 on $C_4 \equiv \langle S_5, \sigma[v \mapsto \delta]\rangle$ where $S_4 \equiv [v := v*\gamma;\ v := v+\beta \parallel v := v+\alpha]$. This problem is similar to Problem 3 but is symbolic, and the commutative plus operator has been moved, so the shape of the graph will be different from Problem 3.

5. Let $C_5 \equiv \langle W, \sigma \rangle$ where $W \equiv$ *while* $x \le n$ *do* $[x := x+1 \;||\; y := y*2]$ *od* and let σ of $x, y,$ and z be $0,\ 1,$ and 2 respectively. Note the parallel construct is in the body of the loop.
   a. Draw an evaluation graph for C_5. (Feel free to to say something like "Let $T \equiv \ldots$" for the loop body, to cut down on the writing.
   b. Draw another evaluation graph for C_5, but this time, use the $\rightarrow^3$ notation to get a straight line graph. Concentrate on the configurations of the form $\langle W, \ldots \rangle$.

6. In $[S_1 \,\|\, S_2 \,\|\, \ldots \,\|\, S_n]$ can any of the threads $S_1, S_2, \ldots, S_n$ contain parallel statements? Can parallel statements be embedded within loops or conditionals?

7. Say we know $\{p_1\}\ S_1\ \{q_1\}$ and $\{p_2\}\ S_2\ \{q_2\}$ under partial or total correctness.
   a. In general, do we know how $\{p_1 \wedge p_2\}\ [S_1 \,\|\, S_2]\ \{q_1 \wedge q_2\}$ will execute? Explain briefly.
   b. What if $p_1 \equiv p_2$? I.e., if we know $\{p\}\ S_1\ \{q_1\}$ and $\{p\}\ S_2\ \{q_2\}$, then do we know how
   $\{p\}\ [S_1 \,\|\, S_2]\ \{q_1 \wedge q_2\}$ will work?
   c. What if in addition, $q_1 \equiv q_2$? I.e., If we know $\{p\}\ S_1\ \{q\}$ and $\{p\}\ S_2\ \{q\}$, do we know how
   $\{p\}\ [S_1 \,\|\, S_2]\ \{q\}$ will work? (This problem is harder)
   d. For parts (a) – (c), does it make a difference if we use $\vee$ instead of $\wedge$?

Solution to Practice 22

Class 22: Basics of Parallel Programs

1. Sequential nondeterministic equivalent of $[x := v \parallel y := v+2 \parallel z := v*2]$:

$$
\begin{array}{l}
\mathit{if}\ T \rightarrow x := v;\ y := v+2;\ z := v*2 \\
\square\ T \rightarrow x := v;\ z := v*2;\ y := v+2 \\
\square\ T \rightarrow y := v+2;\ x := v;\ z := v*2 \\
\square\ T \rightarrow y := v+2;\ z := v*2;\ x := v \\
\square\ T \rightarrow z := v*2;\ x := v;\ y := v+2 \\
\square\ T \rightarrow z := v*2;\ y := v+2;\ x := v \\
\mathit{fi}
\end{array}
$$

2. (Program $[x := 1 \,||\, x := -1]\ ;\ y := y+x]$)

   a. Equivalent sequential nondeterministic program

   $$\mathit{if}\ T \rightarrow x := 1;\ x := -1\ \square\ T \rightarrow x := -1;\ x := 1\ \mathit{fi}$$

   b. Evaluation graph for $\langle\, [x := 1 \,||\, x := -1];\ y := y+x,\ \sigma \,\rangle$

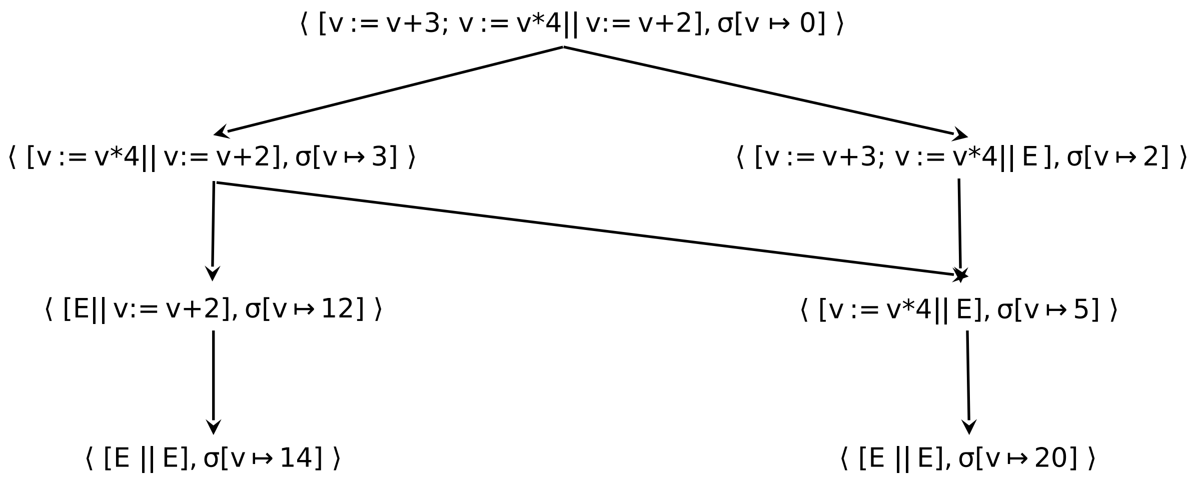
$$
\begin{tikzcd}[column sep=tiny]
& \langle\, [x := 1 \,||\, x := -1]\ ;\ y := y+x,\ \sigma \,\rangle \arrow[dl] \arrow[dr] & \\
\langle\, [E \,||\, x := -1];\ y := y+x,\ \sigma[x \mapsto 1\} \,\rangle \arrow[d] & & \langle\, [x := 1 \,||\, E];\ y := y+x,\ \sigma[x \mapsto -1] \,\rangle \arrow[d] \\
\langle\, [E \,||\, E];\ y := y+x,\ \sigma[x \mapsto -1\} \,\rangle \arrow[d] & & \langle\, [E \,||\, E];\ y := y+x,\ \sigma[x \mapsto 1\} \,\rangle \arrow[d] \\
\langle\, E,\ \sigma[x \mapsto -1\}[y \mapsto \sigma(y)\text{-}1\} \,\rangle & & \langle\, E,\ \sigma[x \mapsto 1\}[y \mapsto \sigma(y)+1\} \,\rangle
\end{tikzcd}
$$

3. (Program $[v := v+3;\ \ v := v*4 \parallel v := v+2]$)

   a. Equivalent sequential nondeterministic program

$$
\begin{array}{l}
\mathit{if}\ T \rightarrow v := v+3;\ \ \mathit{if}\ T \rightarrow v := v*4;\ v := v+2\ \square\ \mathrm{T} \rightarrow v := v+2;\ v := v*4\ \mathit{fi} \\
\square\ T \rightarrow v := v+2;\ v := v+3;\ \ v := v*4 \\
\mathit{fi}
\end{array}
$$

b. Evaluation graph for ⟨ *[v := v+3; v := v*4 || v := v+2],* σ*[v ↦ 0]* ⟩. Note that two of the execution paths happen to merge, so there are only two final states instead of three.

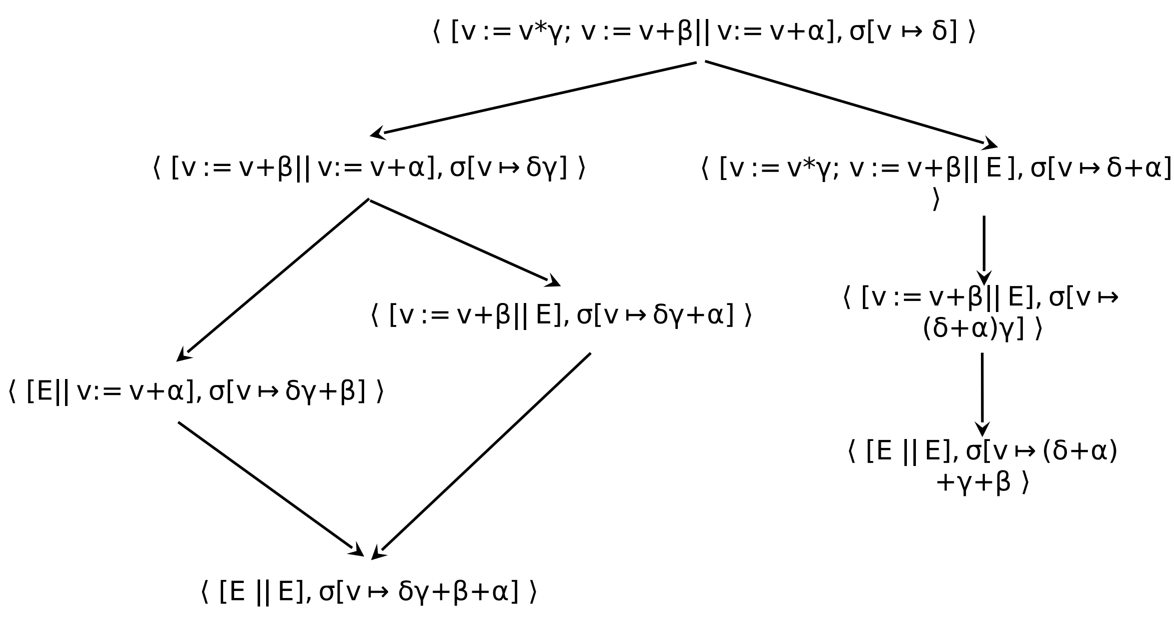


4. (Program *[v := v*γ; v := v+β ‖ v:= v+α]*).

a. Equivalent sequential nondeterministic program

*if T → v := v*γ; if T → v := v+β; v := v+α* ☐ T → *v := v+α; v := v+β fi*
☐ *T → v := v+α; v := v*γ; v := v+β*
fi

b. Evaluation graph for ⟨ *[v := v*γ; v := v+β ‖ v := v+2],* σ*[v ↦ δ]* ⟩



5. (*while* $x \le n$ *do* $[x := x+1 \;||\; y := y*2]$ *od*, if $\sigma(x) = 0$, $\sigma(y) = 1$, and $\sigma(n) = 2$.) Below, let $T \equiv [x := x+1 \;||\; y := y*2]$ (just to cut down on the writing).

   a. A full evaluation graph. Just to be explicit, I wrote $\sigma[x \mapsto 0][y \mapsto 1]$ below but just σ is fine.

$$\begin{tikzcd}
& \langle W, \sigma[x \mapsto 0][y \mapsto 1] \rangle \arrow[d] & \\
& \langle T; W, \sigma[x \mapsto 0][y \mapsto 1] \rangle \arrow[dl] \arrow[dr] & \\
\langle [E \,||\, y:=y*2]; W, \sigma[x \mapsto 1][y \mapsto 1] \rangle \arrow[dr] & & \langle [x:=x+1 \,||\, E]; W, \sigma[x \mapsto 0][y \mapsto 2] \rangle \arrow[dl] \\
& \langle W, \sigma[x \mapsto 1][y \mapsto 2] \rangle \arrow[d] & \\
& \langle T; W, \sigma[x \mapsto 1][y \mapsto 2] \rangle \arrow[dl] \arrow[dr] & \\
\langle [E \,||\, y:=y*2]; W, \sigma[x \mapsto 2][y \mapsto 2] \rangle \arrow[dr] & & \langle [x:=x+1 \,||\, E]; W, \sigma[x \mapsto 1][y \mapsto 4] \rangle \arrow[dl] \\
& \langle W, \sigma[x \mapsto 2][y \mapsto 4] \rangle \arrow[d] & \\
& \langle T; W, \sigma[x \mapsto 2][y \mapsto 4] \rangle \arrow[dl] \arrow[dr] & \\
\langle [E \,||\, y:=y*2]; W, \sigma[x \mapsto 3][y \mapsto 4] \rangle \arrow[dr] & & \langle [x:=x+1 \,||\, E]; W, \sigma[x \mapsto 2][y \mapsto 8] \rangle \arrow[dl] \\
& \langle W, \sigma[x \mapsto 3][y \mapsto 8] \rangle \arrow[d] & \\
& \langle E, \sigma[x \mapsto 3][y \mapsto 8] \rangle &
\end{tikzcd}$$

   b. Evaluation graph abbreviated using $\rightarrow^3$ notation:

$$\langle W, \sigma[x \mapsto 0][y \mapsto 1] \rangle \rightarrow^3 \langle W, \sigma[x \mapsto 1][y \mapsto 2] \rangle \rightarrow^3 \langle W, \sigma[x \mapsto 2][y \mapsto 4] \rangle$$
$$\rightarrow^3 \langle W, \sigma[x \mapsto 3][y \mapsto 8] \rangle \rightarrow \langle E, \sigma[x \mapsto 3][y \mapsto 8] \rangle$$

6. No, in $[S_1 \parallel S_2 \parallel \ldots \parallel S_n]$ the threads cannot contain parallel statements, but yes, parallel statements can be embedded within loops and conditionals.

7. In general, even if $\{p_1\}\ S_1\ \{q_1\}$ and $\{p_2\}\ S_2\ \{q_2\}$ are both valid sequentially, we can't compose them in parallel, even if $p_1 \equiv p_2$ and $q_1 \equiv q_2$. An example is how $\{x > 0\}\ x := x-1\ \{x \geq 0\}$ is valid but $\{x > 0\}\ [x := x-1 \,||\, x := x-1]\ \{x \geq 0\}$ is not. The first $x := x-1$ to execute ends with $x \geq 0$, which is too weak for the second $x := x-1$ to work correctly.

