Weakest Preconditions

Part 2: Calculating wp, wlp; Domain Predicates

CS 536: Science of Programming, Spring 2022

For Problems 2 – 4, just syntactically calculate the *wlp*; don't also logically simplify the result.

2. Calculate the *wlp* in each of the following cases.
   a. $wlp(k := k - s, n = 3 \wedge k = 4 \wedge s = -7)$.
   b. $wlp(n := n*(n-k), n = 3 \wedge k = 4 \wedge s = -7)$.
   c. $wlp(n := n*(n-k)\ ;\ k := k - s, n > k + s)$
3. Let $Q(k, s) \equiv 0 \le k \le n \wedge s = sum(0, k)$ where $sum(u, v)$ is the sum of u, $u+1$, …, v (when $u \le v$) or 0 (when $u > v$).
   a. Calculate $wp(k := k+1; s := s+k , Q(k, s))$.
   b. Calculate $wp(s := s+k+1; k := k+1, Q(k, s))$.
   c. Calculate $wp(s := s+k; k := k+1, Q(k, s))$. (This one isn't compatible with $s = sum(0, k)$.)
4. Calculate the *wp* below.
   a. $wp(\text{if } e \text{ then } \{x := x/2\} \text{ else } \{skip\};\ y := x, x = 5 \wedge y = z)$.
   b. $wp(\text{if } x \ge 0 \text{ then } \{x := x*2\} \text{ else } \{x := y\};\ x := c*x, a \le x < y)$

For Problems 5 and 6, don't forget the domain predicates. You can logically simplify as you go.

5. Calculate p to be the *wp* in $\{p\}\ x := y/b[k]\ \{x > 0\}$.
6. Calculate p_1 and p_2 to be the *wp*'s in $\{p_1\}\ y := sqrt(b[k])\ \{z < y\}$ and $\{p_2\}\ k := x/k\ \{p_1\}$.

Solution to Practice 11 (Weakest Preconditions, pt. 2)

2. (Calculate *wlp*)
   a. $wlp(k := k - s, n = 3 \wedge k = 4 \wedge s = -7) \equiv n = 3 \wedge k - s = 4 \wedge s = -7$
   b. $wlp(n := n*(n-k), n = 3 \wedge k = 4 \wedge s = -7) \equiv n*(n-k) = 3 \wedge k = 4 \wedge s = -7$
   c. $wlp(n := n*(n-k); k := k-s, n > k+s)$
      $\equiv wlp(n := n*(n-k), wlp(k := k-s, n > k+s))$
      $\equiv wlp(n := n*(n-k), n > k-s+s)$
      $\equiv n*(n-k) > k-s+s$

3. (*wp* involving sums) We have $Q(k, s) \equiv 0 \le k \le n \wedge s = sum(0, k)$.
   a. $wp(k := k+1; s := s+k, Q(k, s))$
      $\equiv wp(k := k+1, wp(s := s+k, Q(k, s))$
      $\equiv wp(k := k+1, Q(k, s+k))$
      $\equiv wp(k := k+1, 0 \le k \le n \wedge s+k = sum(0, k))$
      $\equiv 0 \le k+1 \le n \wedge s+k+1 = sum(0, k+1)$

   b. $wp(s := s+k+1; k := k+1, Q(k, s))$
      $\equiv wp(s := s+k+1, wp(k := k+1, Q(k, s))$
      $\equiv wp(s := s+k+1, Q(k+1, s))$
      $\equiv wp(s := s+k+1, 0 \le k+1 \le n \wedge s = sum(0, k+1))$
      $\equiv 0 \le k+1 \le n \wedge s+k+1 = sum(0, k+1)$

   c. $wp(s := s+k; k := k+1, Q(k, s))$
      $\equiv wp(s := s+k, wp(k := k+1, Q(k, s))$
      $\equiv wp(s := s+k, Q(k+1, s))$
      $\equiv wp(s := s+k, 0 \le k+1 \le n \wedge s = sum(0, k+1))$
      $\equiv 0 \le k+1 \le n \wedge s+k = sum(0, k+1)$. Note this isn't compatible with $s = sum(0, k)$.

4. (*wp* of *if-then*)
   a. $wp(\text{if } e \text{ then } \{x := x/2\} \text{ else } \{skip\}; y := x, x = 5 \wedge y = z)$
      $\equiv wp(\text{if } e \text{ then } \{x := x/2\} \text{ else } \{skip\}, wp(y := x, x = 5 \wedge y = z))$
      $\equiv wp(\text{if } e \text{ then } \{x := x/2\} \text{ else } \{skip\}, x = 5 \wedge x = z)$
      $\equiv (e \to wp(x := x/2, x = 5 \wedge x = z)) \wedge (\neg e \to wp(skip, x = 5 \wedge x = z))$
      $\equiv (e \to x/2 = 5 \wedge x/2 = z) \wedge (\neg e \to x = 5 \wedge x = z)$

b. $wp(\text{if } x \ge 0 \text{ then } \{x := x*2\} \text{ else } \{x := y\};\, x := c*x,\ a \le x < y)$.

$\equiv wp(S,\ wp(x := c*x,\ a \le x < y))$ where S is the *if* statement

$\equiv wp(S,\ a \le c*x < y)$

$\equiv wp(\text{if } x \ge 0 \text{ then } \{x := x*2\} \text{ else } \{x := y\},\ a \le c*x < y)$

$\equiv (x \ge 0 \rightarrow wp(x := x*2,\ a \le c*x < y)) \wedge (x < 0 \rightarrow wp(x := y,\ a \le c*x < y))$

$\equiv (x \ge 0 \rightarrow a \le c*(x*2) < y) \wedge (x < 0 \rightarrow a \le c*y < y)$

5. For $\{p\}\ x := y/b[k]\ \{x > 0\}$,

let $p \Leftrightarrow wp(x := y/b[k],\ x > 0)$

$\equiv wlp(x := y/b[k],\ x > 0) \wedge D(x := y/b[k])$

$\equiv y/b[k] > 0 \wedge b[k] \ne 0 \wedge D(b[k])$

$\equiv y/b[k] > 0 \wedge b[k] \ne 0 \wedge 0 \le k < size(b)$

6. For $\{p_1\}\ y := sqrt(b[k])\ \{z < y\}$

let $p_1 \Leftrightarrow wp(y := sqrt(b[k]),\ z < y)$

$\equiv wlp(y := sqrt(b[k]),\ z < y) \wedge D(y := sqrt(b[k]))$

$\equiv z < sqrt(b[k]) \wedge b[k] \ge 0 \wedge D(b[k])$

$\equiv z < sqrt(b[k]) \wedge b[k] \ge 0 \wedge 0 \le D(b[k] < size(b)$

For $\{p_2\}\ k := x/k;\ \{p_1\}$, let

$p_2 \Leftrightarrow wp(k := x/k,\ p_1)$

$\equiv wlp(k := x/k,\ p_1) \wedge D(k := x/k)$

$\equiv p_1[x/k \,/\, k] \wedge k \ne 0$

$\equiv (z < sqrt(b[k]) \wedge b[k] \ge 0 \wedge 0 \le D(b[k] < size(b))\ [x/k \,/\, k] \wedge k \ne 0$

$\equiv z < sqrt(b[x/k]) \wedge b[x/k] \ge 0 \wedge 0 \le D(b[x/k] < size(b) \wedge k \ne 0$