Lectures 1-2: Overview, Propositional and Predicate Logic

CS536 Science of Programming, Fall 2023

Science of Programming

Specifically, Program Verification

Program Verification

this course (mostly)

Formally checking that a program is **correct**

- gives the right answer
- doesn't take too long
- has the right *effects*
- has the right security properties

Usually: that it meets a *specification*

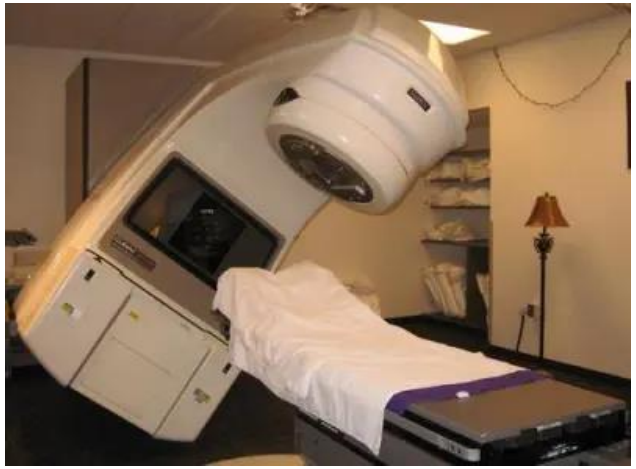
Quick Survey

- How many of you have written a program of > 100 lines of code in the last 6 months?

Testing is not enough

… and it matters a lot:

Boeing 737-MAX
2017-2019



Therac-25 radiation therapy machine
1985-1987

Testing is not enough

Even if you cover all code:

- Unexpected inputs
- Unexpected user behavior
- Concurrency errors (e.g., race conditions)
- Changes in code
- Changes in requirements

Application of verification: Be done with your coding homework!

```
function fib(n: nat): nat
decreases n
{
    if n == 0 then 0 else
    if n == 1 then 1 else
                  fib(n - 1) + fib(n - 2)
}

method ComputeFib(n: nat) returns (r: nat)
    ensures r == fib(n)
{
    var a, b := 0, 1;
    var temp := 0;
    var i := 0;
    while (i < n)
    invariant (a == fib(i)) && (b == fib(i+1)) && (i <= n)
    {
        temp := a + b;
        assert temp == fib(i+2);
        a := b;
        b := temp;
        i := i + 1;
    }
    return a;
}
```

0 Verification Succeeded

Verification Succeeded

Verification isn’t perfect

- Difficult to get right, even for small programs
- Automated tools can help (but then you have to trust those!)
- Have to get the *specification* right

Easy to get specifications wrong

- What should the spec be?

```
//Argument: a list of integers
//Returns: ????
function sort (l: int list)
```

Static types can be seen as a form of verification

- OCaml `sort : int list -> int list`
  - Takes an integer list and returns an integer list.
  - Valid: `sort([8;2;1;6;3]) = [8;2;1;6;3]`
  - Valid: `sort([8;2;1;6;3]) = [10;11;12]`

- Coq
```
sort : forall (l1 : list int), exists (l2: int list),
                Sorted l2 /\ Permutation l1 l2
```
  - Takes an integer list and returns a sorted permutation of it.
  - Valid: `sort([8;2;1;6;3]) = [1;2;3;6;8]`
  - … and nothing else

Static types can be seen as a form of verification

… but that’s a whole other class

Verification: connecting *logical specs* and *formal semantics*

```
function f(int x) {
  if (x > 0) {
    y = x * 2;
  } else {
    y = x * -2;
  }

}
```

x > 0, so y = + * + = +

x <= 0, so y = - * - = +
or y = 0 * - = 0

How do we know that's what this code does?

Well, it's obvious in this case. But not always (or even defined) in complex languages like C

Formal semantics: mathematical description of what code does

Course Outcomes

After taking this class, you should be able to:

- Understand the limits of testing and the importance of verification
- Perform basic verification on programs
- Understand the semantics of programs (including nondeterministic and parallel ones!)

Course Information

- Website: http://cs.iit.edu/~smuller/cs536-f23/
  - Schedule, links, notes
  - Check it frequently!

- Blackboard
  - Download and submit assignments
  - Class recordings

Prerequisites

- Officially: CS331 or CS401 with a min. grade of C
- Informally:
  - Familiarity with basic logic
  - Comfort with mathematics, formal notations
  - *Some* programming experience

  - We'll review some of these concepts quickly today and Wednesday
    - BUT: If you are not at all comfortable with mathematical logic, make sure you learn it this week (not just in class) or consider delaying taking the class

Will there be programming?

- Hard question to answer…
- Will not have to learn a whole new language

- Mostly theory: there will be some proofs
  - Will have to express them formally so they can be checked by computer
  - Proofs *about* programs: we will be using a small language with assignment, if/then/else, while, etc…
  - May have to write some small programs in this small language

Grading

- 40% Homework assignments (every 1-2 weeks)
  - May not be evenly weighted
- 25% Midterm Exam (Tentatively Oct. 23)
- 35% Final Exam

A	B	C	E
90-100	80-89	70-79	<70

- I may curve exam grades depending on the course averages

Exams

- Midterm: 75 minutes, normal class time
- Final exam during finals period (date set by Registrar)
  - Final exam *is* cumulative
- Some kind of notes allowed (past semesters: 1-2 sheets of notes)
- Sections 01, 02: In-person
  - If you CANNOT take the exam in person, let me know
- Section 03: Will get an email discussing options
  - Preferred: take with the in-person students.
  - Also possible: take somewhere else with a proctor

Late Days

- 8 late days per student
- Each late day extends the deadline 24 hours
- Can use <= 2 per assignment
  - Can’t use on exams
- After late days used up: 10% penalty per day late
- No work accepted >2 days late without instructor approval

Academic Honesty

- All submitted work (homework and exams) is to be your own individual work unless specified otherwise.
- Specifically prohibited (but this list isn't exhaustive):
  - Sharing answers with other students
  - Looking online for answers
  - Generative AI (e.g., ChatGPT)
- Specifically permitted:
  - Getting help from TAs or instructor
  - Getting help from the ARC or other official university tutoring resources
    - If you want to use an outside tutor, let me know first

Academic Honesty

- Penalties (for every violation):
  - Zero on the homework or exam
  - Report to academic honesty
    - May result in university-level sanctions after first report

Course Staff

- **Instructor:** Stefan Muller
- **TAs:** Chaoqi Ma, Gagan Beerappa

- Office hours (info including links will be posted):
  - Monday 2pm-3pm SB 218E (Stefan)
  - Tuesday 2pm-3pm SB 004 (Chaoqi)
  - Wednesday 2pm-3pm Google Meet (Gagan)
  - Thursday 10am-11am Zoom (Stefan) AND 2pm-3pm Zoom (Chaoqi)
  - Friday 2pm-3pm Google Meet (Gagan)
- We are here to answer your questions! Really! Yes, all of us!

Other ways to get help

- Discord: IIT CS server, cs536 channel
  - If you're not on it, we'll send an invitation
- Academic Resource Center (ARC): www.iit.edu/arc
  - FREE subject matter tutoring and academic coaching

	Discord	**Office Hours**	**Email**	**ARC**
General questions about lectures, logistics, etc.	✓	✓		
General discussion, clarifications, about HW questions	✓	✓		
Specific questions about your HW answers		✓		✓
More in-depth personal tutoring				✓
Personal matters (accommodations, other requests, etc.)			✓	

Attendance/Sections

- Section 01: In-person
- Section 02: In-person, PhD section
- Section 03: Online (lectures recorded)

- Also:
  - Sections 04, 06: In-person with other instructors
  - Section 05: Online with other instructor

PhD Qualifier Section

- A sufficiently high grade in CS536 meets the requirements for the written qualifier for CS PhDs.
  - **Only if you are in section 02.**
  - If you are taking this class to meet this requirement and are not in section 02, talk to me or switch this ASAP.

  - Section 02 is an in-person section. If you’re a PhD student taking 536 for the qualifier but can’t attend in person, let me know.

Announcements

- Blackboard fixed
- Office Hours times/links are up on Blackboard
  - (times on course website)
- HW1 will be posted today
  - On Blackboard
  - Due 9/7, 11:59 PM (remember: can use up to 2 late days)
  - Submit on Blackboard
  - Start early, make sure you can do it

Syntax and Semantics and Equality

- Syntax: How to write down a "program"
  - Syntactic Equality ($\equiv$): written the same (up to, e.g., parentheses)
  - $2 + 2 - 3 \equiv 2 + 2 - 3 \equiv (2 + 2) - 3$
  - $2 + 2 - 3 \not\equiv 1$
  - $1 + 2 \not\equiv 2 + 1$
- Semantics: What a "program" "means"
  - Semantic Equality ($=$): has the same meaning
  - $2 + 2 - 3 = (2 + 2) - 3 = 4 - 3 = 1$
  - $1 + 2 = 2 + 1$

Propositional Logic

- “Atomic” propositions: variables that can be true (T) or false (F)
  - P, Q
- Connectives: make larger propositions p, q, φ, ψ
  - Negation: $\neg$p (**not** p)
  - Conjunction: p $\land$ q (p **and** q)
  - Disjunction: p $\lor$ q (p **or** q)
  - Conditional: p $\rightarrow$ q (p **implies** q, **if** p **then** q)
  - Biconditional: p $\leftrightarrow$ q (p **iff** q, p **if and only if** q)
- Precedence (order of operations): in the above order

$$p \land q \rightarrow r \lor \neg q \leftrightarrow s \;\equiv [(p \land q) \rightarrow \big(r \lor (\neg q)\big)] \leftrightarrow s$$

The semantics of a proposition are their truth values in different states

State σ: Assignment of truth values (T, F) to proposition variables

Written, e.g., $\{P = T, Q = F\}$

Only one assignment per variable: ~~$\{P = T, P = F\}$~~

Only assigns to variables: ~~$\{P \vee Q = T\}$~~

A state fitting these requirements is *well-formed* (opp. *ill-formed*)

$\sigma \vDash p$: p “satisfied” (true) in state σ

Truth value of propositions determined by truth tables

P	Q	¬P	P ∧ Q	P ∨ Q	P → Q	P ↔ Q
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

- ∧, ∨ are commutative and associative: $P \wedge Q = Q \wedge P \quad P \wedge (Q \wedge R) = (P \wedge Q) \wedge R$
- → is *not* commutative *or* associative: $F \to T \neq T \to F \quad (F \to T) \to F \neq F \to (T \to F)$
- ↔ is commutative and associative: $P \leftrightarrow Q = Q \leftrightarrow P \quad (P \leftrightarrow Q) \leftrightarrow R = P \leftrightarrow (Q \leftrightarrow R)$

Some more facts about conditionals

For a conditional $P \rightarrow Q$:

- The *inverse* $\neg P \rightarrow \neg Q$ *does not have the same* truth value
- The *converse* $Q \rightarrow P$ *does not have the same* truth value
  - (But is the same as the inverse)
- The *contrapositive* $\neg Q \rightarrow \neg P$ has the *same* truth value

To determine truth value, a state needs to be *proper* for the proposition

- Proper: defines truth values for all variables in the proposition

Proposition: $P \wedge Q \rightarrow R \vee \neg Q \leftrightarrow S$

Proper:
- $\{P = T, Q = F, R = F, S = T\}$
- $\{Q = T, P = F, S = F, R = T\}$
- $\{Q = T, P = F, S = F, R = T, T = F\}$

Improper:
- $\{P = T, Q = F, R = F\}$
- $\{P = F, S = F\}$

For a well-formed and proper state, a proposition is satisfied or unsatisfied

- $\{P = T; Q = F; R = F\} \vDash (P \wedge Q) \rightarrow R?$
- $\{P = T; Q = F; R = T\} \vDash (P \wedge Q) \rightarrow R?$
- $\{P = T; Q = F; R = F\} \vDash (P \vee Q) \rightarrow R?$

A proposition can be a *tautology*, *contradiction* or *contingency*

Assume σ is well-formed and proper

- Tautology: $\sigma \vDash p$ for all σ (Also write just $\vDash p$)
- Contradiction: $\sigma \nvDash p$ for all σ (Equivalent: $\vDash \neg p$)
  - Note that this is *not* the same as $\nvDash p$ (that just says *p* is not a tautology)
- Contingency: There exist σ_1 and σ_2 such that $\sigma_1 \vDash p$ and $\sigma_2 \nvDash p$
  (Equivalent: $\nvDash \neg p$ and $\nvDash p$)

Logical implication $\Rightarrow$

$P \Rightarrow Q$ if whenever P is true, so is Q ("P implies Q")

Note: this is not the same as $P \rightarrow Q$:

- They're related: $P \Rightarrow Q$ means $\vDash P \rightarrow Q$
- We write $\rightarrow$ *in* propositions, we use $\Rightarrow$ to talk *about* propositions
  - (like how we put + in mathematical expressions, we use = to talk about them)

Logical equivalence $\Leftrightarrow$

$P \Leftrightarrow Q$ means $P \Rightarrow Q$ and $Q \Rightarrow P$

Semantic equality (=) on logical propositions

Some useful facts

Commutativity $p \vee q \Leftrightarrow q \vee p$

$p \wedge q \Leftrightarrow q \wedge p \quad (p \leftrightarrow q) \Leftrightarrow (q \leftrightarrow p)$

Associativity $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$

$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$

Distributivity/Factoring

$(p \vee q) \wedge r \Leftrightarrow (p \wedge r) \vee (q \wedge r)$

$(p \wedge q) \vee r \Leftrightarrow (p \vee r) \wedge (q \vee r)$

Transitivity [Note: $\Rightarrow$, not $\Leftrightarrow$ here]

$(p \rightarrow q) \wedge (q \rightarrow r) \Rightarrow (p \rightarrow r)$

$(p \leftrightarrow q) \wedge (q \leftrightarrow r) \Rightarrow (p \leftrightarrow r)$

Identity: $p \wedge T \Leftrightarrow p$ and $p \vee F \Leftrightarrow p$

Idempotentcy: $p \vee p \Leftrightarrow p$ and $p \wedge p \Leftrightarrow p$

Domination: $p \vee T \Leftrightarrow T$ and $p \wedge F \Leftrightarrow F$

Absurdity: $(F \rightarrow p) \Leftrightarrow T$

Contradiction: $p \wedge \neg p \Leftrightarrow F$

Excluded middle: $p \vee \neg p \Leftrightarrow T$

Double negation: $\neg\neg p \Leftrightarrow p$

DeMorgan's Laws $\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$

$\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$

Defn. of $\rightarrow$ *and* $\leftrightarrow$ $(p \rightarrow q) \Leftrightarrow (\neg p \vee q)$

$(p \leftrightarrow q) \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

Commutativity $p \vee q \Leftrightarrow q \vee p$

$p \wedge q \Leftrightarrow q \wedge p \quad (p \leftrightarrow q) \Leftrightarrow (q \leftrightarrow p)$

Associativity $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$

$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$

Distributivity/Factoring

$(p \vee q) \wedge r \Leftrightarrow (p \wedge r) \vee (q \wedge r)$

$(p \wedge q) \vee r \Leftrightarrow (p \vee r) \wedge (q \vee r)$

Transitivity [Note: $\Rightarrow$, not $\Leftrightarrow$ here]

$(p \rightarrow q) \wedge (q \rightarrow r) \Rightarrow (p \rightarrow r)$

$(p \leftrightarrow q) \wedge (q \leftrightarrow r) \Rightarrow (p \leftrightarrow r)$

Identity: $p \wedge T \Leftrightarrow p$ and $p \vee F \Leftrightarrow p$

Idempotentcy: $p \vee p \Leftrightarrow p$ and $p \wedge p \Leftrightarrow p$

Domination: $p \vee T \Leftrightarrow T$ and $p \wedge F \Leftrightarrow F$

Absurdity: $(F \rightarrow p) \Leftrightarrow T$

Contradiction: $p \wedge \neg p \Leftrightarrow F$

Excluded middle: $p \vee \neg p \Leftrightarrow T$

Double negation: $\neg\neg p \Leftrightarrow p$

DeMorgan's Laws $\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$

$\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$

Defn. of $\rightarrow$ *and* $\leftrightarrow$ $(p \rightarrow q) \Leftrightarrow (\neg p \vee q)$

$(p \leftrightarrow q) \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

$$\neg(p \rightarrow q) \Rightarrow (p \wedge \neg q)$$

Commutativity $p \lor q \Leftrightarrow q \lor p$

$p \land q \Leftrightarrow q \land p \quad (p \leftrightarrow q) \Leftrightarrow (q \leftrightarrow p)$

Associativity $(p \lor q) \lor r \Leftrightarrow p \lor (q \lor r)$

$(p \land q) \land r \Leftrightarrow p \land (q \land r)$

Distributivity/Factoring

$(p \lor q) \land r \Leftrightarrow (p \land r) \lor (q \land r)$

$(p \land q) \lor r \Leftrightarrow (p \lor r) \land (q \lor r)$

Transitivity [Note: $\Rightarrow$, not $\Leftrightarrow$ here]

$(p \rightarrow q) \land (q \rightarrow r) \Rightarrow (p \rightarrow r)$

$(p \leftrightarrow q) \land (q \leftrightarrow r) \Rightarrow (p \leftrightarrow r)$

Identity: $p \land T \Leftrightarrow p$ and $p \lor F \Leftrightarrow p$

Idempotentcy: $p \lor p \Leftrightarrow p$ and $p \land p \Leftrightarrow p$

Domination: $p \lor T \Leftrightarrow T$ and $p \land F \Leftrightarrow F$

Absurdity: $(F \rightarrow p) \Leftrightarrow T$

Contradiction: $p \land \neg p \Leftrightarrow F$

Excluded middle: $p \lor \neg p \Leftrightarrow T$

Double negation: $\neg\neg p \Leftrightarrow p$

DeMorgan's Laws $\neg(p \land q) \Leftrightarrow (\neg p \lor \neg q)$

$\neg(p \lor q) \Leftrightarrow (\neg p \land \neg q)$

Defn. of $\rightarrow$ and $\leftrightarrow$ $(p \rightarrow q) \Leftrightarrow (\neg p \lor q)$

$(p \leftrightarrow q) \Leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$

$((r \rightarrow s) \land r) \Rightarrow s$ ("Modus ponens")

$T \Rightarrow P \land \neg (Q \land R) \text{ -> } ((Q \land R) \text{ -> } \neg P)$

Announcements

- New TA: Param Modi
  - Office Hours: Tuesday/Thursday 11:30am-12:30pm (Online)

- HW1 is up on Blackboard.
  - Covers material through part of today’s lecture

More facts

- If $\vDash p$ and $\vDash q$ then $\vDash p \wedge q$
- If $\vDash p$ then $\vDash p \vee q$ and $\vDash q \vee p$ (for any q)
- If $\vDash p \wedge q$ then $\vDash p$ and $\vDash q$
- If $\vDash p \rightarrow r$ and $\vDash q \rightarrow r$ and $\vDash p \vee q$ then $\vDash r$

Predicate (First-Order) Logic extends Prop. Logic with values in a domain

- (e.g. the integers)

- We'll also use variables like *x* to hold integers (or values of whatever domain we're using)
- Predicate: a function from values or variables in the domain to T or F
  - e.g. isEven(x), Greater(x, 0), Greater(x, y)

    "Syntactic sugar": x >0, x > y

We can use predicates with all the existing connectives

- Greater(x, y) $\vee$ Greater(y, x) $\vee$ Equal(x, y)
- Greater(x, y) $\vee$ Equal(x, y)
- Greater(x, y) $\rightarrow$ Greater(x, y) $\vee$ Equal(x, y)
- Greater(x, y) $\vee$ Equal(x, y) $\leftrightarrow$ $\neg$ Greater(y, x)

States can now have integer vars too

- $\{x = 5, y = 5\} \vDash \text{Greater}(x, y) \lor \text{Equal}(x, y)$
- $\{x = 4, y = 5\} \nvDash \text{Greater}(x, y) \lor \text{Equal}(x, y)$
- $\{x = 5, y = 5, P = T\} \vDash (\text{Greater}(x, y) \lor \text{Equal}(x, y)) \land P$

Quantifiers introduce variables

- $\forall x \in \mathbb{Z}. p$ (**for all** x, p)
- $\exists x \in \mathbb{Z}. p$ (**there exists** x **such that** p)
- (may omit the domain if clear)

- $\vDash \forall x. \forall y. \text{Greater}(y, x) \vee \text{Greater}(x, y) \vee \text{Equal}(x, y)$
- $\vDash \forall x. \forall y. \text{Greater}(x, y) \vee \text{Equal}(x, y) \ \leftrightarrow \neg\text{Greater}(y, x)$
- $\vDash \forall x. \exists y. \text{Greater}(y, x)$
- $\vDash \neg\exists x. \text{Greater}(x, 2) \wedge \text{isPrime}(x) \wedge \text{isEven}(x)$

Equivalence with quantifiers gets a little tricky

- $\forall x. P(x) = \forall y. P(y)$ because $\forall x. P(x) \Leftrightarrow \forall y. P(y)$

- Is $\forall x. P(x) \equiv \forall y. P(y)$?
  - For now, let's say no.
  - But there are good reasons to consider them equivalent in more-than-just-semantic ways. We may discuss this later.

DeMorgan's Laws for quantifiers

- $\neg \exists x. P \Leftrightarrow \forall x. \neg P$
- $\neg \forall x. P \Leftrightarrow \exists x. \neg P$

$$\neg \exists x. \text{Greater}(x, 2) \wedge \text{isPrime}(x) \wedge \text{isEven}(x)$$
$$\Leftrightarrow \forall x. \neg \text{Greater}(x, 2) \vee \neg \text{isPrime}(x) \vee \neg \text{isEven}(x)$$

$$\forall x. \exists y. \text{Greater}(y, x) \Leftrightarrow \neg \exists x. \forall y. \neg \text{Greater}(y, x)$$

- How would we actually go about proving $\models \forall x. \exists y. \mathrm{Greater}(y, x)$?
- Formal systems for this kind of proof are complicated (we'd need to know the semantics of Greater), but here's an idea:

- To prove $\models \forall x. p(x)$, p(x) must hold *regardless* of the choice of *x*.
- To prove $\models \exists x. p(x)$, come up with a *witness*: a value of x such that p(x) holds.

Examples

$\forall x \in \mathbb{Z} .\ x \neq 0 \rightarrow x \leq x^2$	True
$\exists x \in \mathbb{Z} .\ x \neq 0 \wedge x \geq x^2$	True (use 1 as a witness)
$x > 0 \rightarrow \exists\, y .\ y^2 < x$	Tautology (0 works as a witness regardless of choice of x)
$x > 0 \rightarrow y^2 < x$	Contingency
$\exists y .\ (y < 0 \wedge y > x^2)$	Contradiction (false for every choice of x)

We can define our own predicates

- e.g. Positive(x) = Greater(x, 0) $\wedge \neg$Equal(x, 0)

- The body should be a proposition over the parameters to the predicate function.
- e.g. **not** square(x) = x * x
- but: square(x, y) = (y = x * x)

Predicates should be simple

- For an array a, AllPositive(a, m, n), should mean that a[m], …, a[n] are all positive.

- First try: AllPositive(a, m, n) = Positive(a[m]) ∧ ... ∧ Positive(a[n])
- Second try: maybe a loop?
- Fine in a regular programming language, but the purpose of our predicates is debugging programs
  - No point if our predicates are as hard to debug as the programs!

- AllPositive(a, m, n) = $\forall i, (m \leq i \wedge i \leq n) \rightarrow \text{Positive}(a[i])$

Sorted as a predicate

- Sorted(a, m, n): a[m], …, a[n] are in sorted order
  - (i.e. a[m] ≤ a[m+1] ≤ … ≤ a[n])
  - (i.e., a[m] ≤ a[m+1] and a[m+1] ≤ a[m+2] and…)
  - (i.e., for all i, a[i] ≤ a[i+1])

- $\mathrm{Sorted}(a, m, n) = \forall i, (m \leq i \land i < n) \rightarrow a[i] \leq a[i+1]$