Dynamics and Statics

Examples

Each step is a derivation

$$\dfrac{\dfrac{\dfrac{}{\overline{1}+\overline{2} \mapsto \overline{3}}\ \text{S-1}}{(\overline{1}+\overline{2})+\overline{3} \mapsto \overline{3}+\overline{3}}\ \text{S-4}}{\overline{0}+((\overline{1}+\overline{2})+\overline{3}) \mapsto \overline{0}+(\overline{3}+\overline{3})}\ \text{S-5}$$

"Search rules" – find the part of the exp. that can step

Usually don't write the full derivation

$$\begin{aligned}
& \overline{0}+((\overline{1}+\overline{2})+\overline{3}) \\
\mapsto\ & \overline{0}+(\overline{3}+\overline{3}) \quad \text{> Also its own derivation...} \\
\mapsto\ & \overline{0}+\overline{6} \\
\mapsto\ & \overline{6}
\end{aligned}$$

$$\begin{aligned}
& |\text{"ab"} \wedge \text{"c"}| \\
\mapsto\ & |\text{"abc"}| \\
\mapsto\ & 3
\end{aligned}$$

$e \mapsto^n e'$ "e evaluates to e' in n steps"

$$\dfrac{}{e \mapsto^0 e}\ (1) \qquad \dfrac{e \mapsto e' \quad e' \mapsto^n e''}{e \mapsto^{n+1} e''}\ (2)$$

$e \mapsto^* e'$ "e evaluates to e' in 0 or more steps"

$$\dfrac{}{e \mapsto^* e}\ (3) \qquad \dfrac{e \mapsto e' \quad e' \mapsto^* e''}{e \mapsto^* e''}\ (4)$$

Thm: $e \mapsto^* e'$ if and only if $e \mapsto^n e'$ for some $n \geq 0$

Proof.

$\Rightarrow$ By induction on the derivation of $e \mapsto^* e'$

Case (3) Then $e = e'$. By (1), $e \mapsto^0 e$

Case (4) Then $e \mapsto e''$ and $e'' \mapsto^* e'$.

By induction, $e'' \mapsto^n e'$ for some $n \geq 0$.

By (2), $e \mapsto^{n+1} e'$, and $n \geq 0$.

$\Leftarrow$ By induction on the derivation of $e \mapsto^n e'$.

Case (1) Then $n = 0$ and $e = e'$. By (3), $e \mapsto^* e$.

Case (2) Then $n = m+1$ and $e \mapsto e''$ and $e'' \mapsto^m e'$.

By induction, $e'' \mapsto^* e'$. By (4), $e \mapsto^* e'$. $\square$

Ex. $1 + \text{"Hello"} \mapsto ?$

$(1+2) + \text{"Hello"} \mapsto 3 + \text{"Hello"} \mapsto ?$

<u>Static Semantics</u> (Type system)

Syntax for types: $\tau ::= \text{int} \mid \text{string}$

Judgment: $e : \tau$ "e has type τ"

$$\frac{}{\overline{n} : \text{int}}\ (T\text{-}1) \qquad \frac{}{\text{"s"} : \text{string}}\ (T\text{-}2) \qquad \frac{e_1 : \text{int} \quad e_2 : \text{int}}{e_1 + e_2 : \text{int}}\ (T\text{-}3)$$

$$\frac{e_1 : \text{string} \quad e_2 : \text{string}}{e_1 \mathbin{\hat{}} e_2 : \text{string}}\ (T\text{-}4) \qquad \frac{e : \text{string}}{|e| : \text{int}}\ (T\text{-}5)$$

These need to be e's!
Not enough to just have $\overline{n}$,
then <u>we</u> couldn't give a type
to $(\overline{1}+\overline{2})+\overline{3}$

Typing derivations

$$\dfrac{\dfrac{}{\overline{0}:\text{int}}(T\text{-}1) \qquad \dfrac{\dfrac{\dfrac{}{\overline{1}:\text{int}}(T\text{-}1) \quad \dfrac{}{\overline{2}:\text{int}}(T\text{-}1)}{\overline{1}+\overline{2}:\text{int}}(T\text{-}3) \qquad \dfrac{}{\overline{3}:\text{int}}(T\text{-}1)}{(\overline{1}+\overline{2})+\overline{3}:\text{int}}}{\overline{0}+((\overline{1}+\overline{2})+\overline{3}):\text{int}}(T\text{-}3)$$

$$\dfrac{\dfrac{\dfrac{}{\text{"ab"}:\text{string}}(T\text{-}2) \quad \dfrac{}{\text{"c"}:\text{string}}(T\text{-}2)}{\text{"ab"} \wedge \text{"c"}:\text{string}}(T\text{-}4)}{|\text{"ab"} \wedge \text{"c"}|:\text{int}}(T\text{-}5)$$

$1 + \text{"Hello"} \not: \tau$ for any τ.

Idea: If $e:\tau$, then e will never "get stuck".