Parallelism and Concurrency

Parallelism: Evaluate on multiple processors to speed up computation
Concurrency: Use multiple threads sharing resources (may/may not be parallel)

$e ::= \ldots \mid e \,||\, e$

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1 || e_2 : \tau_1 \times \tau_2} \qquad \frac{e_1 \mapsto e_1'}{e_1 || e_2 \mapsto e_1' || e_2} \qquad \frac{e_2 \mapsto e_2'}{e_1 || e_2 \mapsto e_1 || e_2'}$$

"interleaving"

$$\frac{e_1 \text{ val} \quad e_2 \text{ val}}{e_1 || e_2 \mapsto (e_1, e_2)}$$

Why not $\dfrac{e_1 \mapsto e_1' \quad e_2 \mapsto e_2'}{e_1 || e_2 \mapsto e_1' || e_2'}$?

$1+2 \,||\, 3+4 \mapsto 3 \,||\, 3+4 \mapsto 3 \,||\, 7$
$\mapsto 1+2 \,||\, 7 \mapsto 3 \,||\, 7$ } same answer! (always true for STLC)

$$\frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2}{e_1 || e_2 \Downarrow (v_1, v_2)}$$ - can't capture diff. interleavings but that's ok.

"Nested" parallelism

```
fix fib = λn. if n ≤ 1 then n
              else
                let p = fib (n-1) || fib (n-2)
                in
                (fst p) + (snd p)
```

What about IMP?

$S ::= x := e \mid \text{if } e \text{ then } s \text{ else } s \text{ fi} \mid \text{while } e \text{ do } s \text{ od}$
$\mid s; s \mid \text{skip} \mid s \| s$

$$\frac{\langle s_1, \sigma\rangle \mapsto \langle s_1', \sigma'\rangle}{\langle s_1 \| s_2, \sigma\rangle \mapsto \langle s_1' \| s_2, \sigma'\rangle} \qquad \frac{\langle s_2, \sigma\rangle \mapsto \langle s_2', \sigma'\rangle}{\langle s_1 \| s_2, \sigma\rangle \mapsto \langle s_1 \| s_2', \sigma'\rangle}$$

$$\frac{}{\langle \text{skip} \| \text{skip}, \sigma\rangle \mapsto \langle \text{skip}, \sigma\rangle}$$

$\langle x := x+1 \| x := x*2, \{x=1\}\rangle \mapsto^* \langle \text{skip} \| x := x*2, \{x=2\}\rangle \mapsto^* \langle \text{skip}, \{x=4\}\rangle$
$\mapsto^* \langle x := x+1 \| \text{skip}, \{x=2\}\rangle \mapsto^* \langle \text{skip}, \{x=3\}\rangle$?!

Except there are also more!

$\langle x := x+\overline{1} \| x := x*\overline{2}, \{x=1\}\rangle$
$\mapsto \langle x := \overline{1}+\overline{1} \| x := x*\overline{2}, \{x=1\}\rangle$
$\mapsto \langle x := \overline{1}+\overline{1} \| x := \overline{1}+\overline{2}, \{x=1\}\rangle$
$\mapsto^* \langle x := \overline{2} \| x := \overline{2}, \{x=1\}\rangle$
$\mapsto^* \langle \text{skip}, \{x=2\}\rangle$

$\langle (\text{while } x \text{ do skip od}) \| x := 1, \{x=0\}\rangle$
$\mapsto^* \ldots$ or $\mapsto^* \ldots$ (forever)
$\mapsto^* \langle \text{while } x \text{ do skip od} \| \text{skip}, \{x=1\}\rangle$
$\mapsto^* \langle \text{skip}, \{x=1\}\rangle$