| | |
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| Name | CWID |

# Quiz
# 1

## Sept 24th, 2018
## Due Oct 2nd, 11:59pm

## Quiz 1: CS525 - Advanced Database Organization

## Results

*Please leave this empty!*

| 1.1 | 1.2 | 1.3 | 1.4 | Sum |
|---|---|---|---|---|
| | | | | |

# Instructions

- **You have to hand in the assignment using your blackboard**
- **This is an individual and not a group assignment**
- Multiple choice questions are graded in the following way: You get points for correct answers and points subtracted for wrong answers. The minimum points for each questions is **0**. For example, assume there is a multiple choice question with 6 answers - each may be correct or incorrect - and each answer gives 1 point. If you answer 3 questions correct and 3 incorrect you get 0 points. If you answer 4 questions correct and 2 incorrect you get 2 points. ...
- For your convenience the number of points for each part and questions are shown in parenthesis.
- There are 4 parts in this quiz
  1. SQL
  2. Relational Algebra
  3. Index Structures
  4. Result Size Estimation

## Part 1.1 SQL (Total: 31 + 10 bonus points Points)

Consider the following political party schema and example instance. The example data should not be used to formulate queries. SQL statements that you write should return the correct result for every possible instance of the schema!

### party

| pName | leaning | established | endowment |
|---|---|---|---|
| JDF | middle right | 1950 | 6,000,000 |
| LKE | middle left | 1951 | 5,500,000 |
| HHH | far right | 2010 | 2,300,000 |
| GGK | middle | 1980 | 10,000,000 |

### member

| name | party | since |
|---|---|---|
| Peter | JDF | 2014 |
| Bob | JDF | 2014 |
| Alice | JDF | 2011 |
| Alice | GGK | 1990 |

### positions

| name | pos | year | salary |
|---|---|---|---|
| Alice | Governor of Alaska | 2017 | 100,000 |
| Alice | Vice President | 2018 | 200,000 |
| Bob | Secretary of State | 2016 | 150,000 |

### donations

| pName | year | donor | amount |
|---|---|---|---|
| JDF | 2017 | Walmart | 20,000,000 |
| HHH | 2018 | Koch | 40,000,000 |

**Hints:**

- Attributes with black background are the primary key attributes of a relation
- Attribute `party` of relation `member` is a foreign keys to relation `party`.
- Attribute `pName` of relation `donations` is a foreign key to relation `party`.

## Question 1.1.1 (2 Points)

Write an SQL query that returns for each party the total amount of donation dollars per year as a rolling sum, i.e., the output for 2016 will be the sum of all of the donations made in up to and including year 2016.

## Solution

```
SELECT DISTINCT p.pName,
                year,
                sum(amount) OVER (PARTITION BY p.pName
                                  ORDER BY year ASC)
                AS totalDon
FROM party p JOIN donations d ON (p.pName = d.pName)
```

## Question 1.1.2 (4 Points)

For each political leaning of parties (attribute `leaning`), return the names of the three donors which have donated the largest total donation (sum of donated dollars) to parties with this leaning.

## Solution

```
WITH totalDon AS (
    SELECT leaning, donor, sum(amount) AS ttl
    FROM party p NATURAL JOIN donations d
    GROUP BY leaning, donor
  ),
  ranking AS (
    SELECT leaning, donor,
           row_number() OVER (PARTITION BY leaning
                              ORDER BY ttl DESC) AS rank
    FROM totalDon
  )
SELECT leaning, donor
FROM ranking
WHERE rank <= 3
```

## Question 1.1.3 (4 Points)

Write a query that calculates the average increase of salaries over positions over the years. That is for year, you have to compute the average factor by which the average salaries of positions have increased (or decreased) compared to the previous year. For instance, assuming we have only two positions $A$ and $B$ and in 2014 the average salary for position $A$ is 20000 and in 2015 it is 40000, then the average pay for this position has increased by a factor of 2.0.

## Solution

```
WITH posAvg AS (
     SELECT avg(salary) AS avgSal, pos, year
     FROM positions
     GROUP BY pos, year
  ),
  peryearIncr AS (
     SELECT DISTINCT pos, year, avgSal - (first_value(avgSal)
                                            OVER (PARTITION BY pos
                                                  ORDER BY year ASC
                                                  ROWS BETWEEN 1 PRECEDING
                                                       AND CURRENT ROW) AS incr
     FROM posAvg
  )
SELECT year, avg(incr) AS avgIncr
FROM peryearIncr
GROUP BY year
```

## Question 1.1.4 (5 Points)

Write an SQL query that returns positions (attribute pos) which have been held by some members of every party. That is, for each party we can find a member that has held the position.

## Solution

```
SELECT DISTINCT pos
FROM positions p
WHERE NOT EXISTS (SELECT pName
                  FROM party y
                  WHERE NOT EXISTS (SELECT *
                                    FROM member m, positions p2
                                    WHERE m.party = y.pName
                                          AND m.name = p2.name
                                          AND p2.pos = p.pos)
```

of course the aggregation + comparison version also works!

## Question 1.1.5 (2 Points)

Write an SQL Query that returns party members (their `name`) that have not have held any positions with a salary larger than $200,000.

## Solution

```
SELECT name
FROM members
WHERE name NOT IN (SELECT name FROM positions WHERE salary > 200000)
```

## Question 1.1.6 (2 Points)

Write an SQL query that returns the names of donors that have donated more than 50,000,000 in total.

## Solution

```
SELECT donor
FROM donations
GROUP BY donor
HAVING sum(amount) > 50000000
```

## Question 1.1.7 (5 Points)

Write an SQL query that returns parties whose members have each hold at least one position.

## Solution

```
WITH memInter AS (
     SELECT name, party,
            since AS start,
            DECODE(last_value() OVER (PARTITION BY name
                                      ORDER BY since ASC
                                      ROWS BETWEEN CURRENT ROW AND 1 FOLLOWING),
                   9999) AS end -- any other large enough value would work too
     FROM member
   ),
   memPos AS (
     SELECT name, party
     FROM memInter m,  p
     WHERE m.name = p.name AND p.year BETWEEN m.start AND m.end
   )
SELECT party
FROM members m
GROUP BY party
HAVING count(DISTINCT name) = (SELECT count(DISTINCT name)
                               FROM memPos
                               WHERE m2.party = m.party)
```

## Question 1.1.8 (4 Points)

Write an SQL query that returns pairs of persons (party members) that are members of the same set of parties, e.g., Peter and Alice are both members of JDF, but Alice is also a member of GGK which Bob is not. Thus, the pair `(Alice,Bob)` should not be returned. Ensure that a pair of users is only returned once (e.g., do not both return `(Peter,Bob)` and `(Bob,Alice)`).

## Solution

```
WITH samePart AS (
     SELECT m1.name AS name1, m2.name AS name2, count(DISTINCT m1.party) AS numP
     FROM member m1, member m2
     WHERE m1.party = m2.party AND m1.name < m2.name
     GROUP BY m1.name, m2.name
  ),
  memParties AS (
     SELECT m.name, count(*) AS  numP
  )
SELECT name1, name2
FROM samePart, memParties m
WHERE name1 = m1.name AND name2 = m2.name AND m.numP = m.numP
```

## Question 1.1.9 (4 Points)

Write a query that returns for each party and each year, the total amount of donations (dollar amount) the party has received in that year plus the two previous years. For instance, the result for a party in 2014 should include all donations made in 2012-2014 to that party.

## Solution

```
SELECT DISTINCT year, pName,
       sum(amount) OVER (PARTITION BY pName ORDER BY year
                         ROWS BETWEEN 2 PRECEDING AND CURRENT ROW) AS lastThree
FROM donations;
```

## Question 1.1.10 Optional Bonus Question (10 Bonus Points)

Let's prove that SQL with recursion and arithmetic expressions is turing complete. To do that we are going to write a simulator for turing machines as a recursive SQL query. Recall that a turing machine is a tuple $(Q, \Gamma, b, \Sigma, q_0, F, \delta)$ where $Q$ is a set of states, $\Gamma$ is tape alphabet, $b \in Gamma$ is the "blank" symbol, $\Sigma \subseteq \Gamma$ is the input alphabet, $q_0 \in Q$ is the initial state, $F \subseteq \Gamma$ is the set of final states, and $\delta : (Q - F) \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$ is the transition function. The state of a turing machine is stored on an infinite tape consisting of cells indexed by position. Each position of the tape stores a symbol from $\Gamma$. The initial state of the tape is the input to the turing machine. Only finitely many cells on the initial tape state may be non-blank (unequal to $b$). The turing machine starts in state $q_0$ at position 0 of the tape. In each step of its computation it reads the symbol under the current position of the tape and applies $\delta$ taking its current state and the symbol read from the tape as input. The result of $\delta$ is the new state for the next state, the symbol to be written to the tape at the current position, and the direction to move on the tape (if $L$ then we update $pos = pos - 1$ and $pos = pos + 1$ otherwise). The computation of the turing machine stops once the computation reaches a final state $q_f$, i.e., a state in the set $F$.

We encode the turing machine and its tape using the following schema:

- `states(q)` - this relation stores the states of the turing machine, i.e., it encodes $Q$
- `final(q)` - this relation stores $F$
- We denote the start state by the string "q0".
- `delta(qin, sin, qnew, sout, dir)` - This relation encodes $\delta$. If $\delta((q, s)) = (q', s', d)$ then this would be encoded as a tuple `(q,s,q',s',d)`.
- `tape(pos,sym)` - This relation stores the symbol stored at each position of the initial configuration of the tape (the input). Positions that store the blank symbol are omitted from the relation.
- For simplicity assume that the blank symbol is the character *'b'* and any symbol from $\Gamma$ can be encoded as a single character

Write a single recursive SQL query that simulates the turing machine. The output of the query should be the final configuration of the tape once the machine reaches one of the final states from $F$ (if the machine halts on this particular input). An example input database and result (tape state) is shown below.

**states**

| q |
|---|
| q0 |
| q1 |
| q2 |

**final**

| q |
|---|
| q2 |

**delta**

| qin | sin | qnew | sout | dir |
|---|---|---|---|---|
| q0 | 0 | q1 | 1 | R |
| q0 | 1 | q2 | 1 | R |
| q0 | b | q2 | b | R |
| q1 | 0 | q1 | 1 | R |
| q1 | 1 | q2 | 1 | R |
| q1 | b | q2 | b | R |

**tape**

| pos | sym |
|---|---|
| 0 | 0 |
| 1 | 0 |
| 2 | 0 |
| 3 | 1 |

**result**

| pos | sym |
|---|---|
| 0 | 1 |
| 1 | 1 |
| 2 | 1 |
| 3 | 1 |

## Solution

Code written for Postgres and uses Postgres's array datatype and array aggregation function. The problem can also be solved without arrays, but this requires the use of window functions.

```
WITH RECURSIVE ntape(pos,state,iter,tape) AS (
   SELECT DISTINCT 1 AS pos, 'q0' AS state, 0 AS iter,
                  array_agg(sym) OVER (ORDER BY pos
                                       ROWS BETWEEN UNBOUNDED PRECEDING
                                            AND UNBOUNDED FOLLOWING) AS tape
   FROM tape
UNION ALL
      SELECT CASE WHEN qnew IN (SELECT * FROM final) THEN pos
                  WHEN d.dir = 'R' THEN pos + 1
                  ELSE pos-1 END AS pos,
         d.qnew AS state,
             iter + 1 AS iter,
             n.tape[1:n.pos-1] || d.sout || n.tape[n.pos+1:] AS tape
      FROM ntape n, delta d
      WHERE d.qin = n.state
            AND (d.sin = n.tape[pos]) OR (n.tape[pos] IS NULL AND d.sin='b')
            AND d.qin NOT IN (SELECT * FROM final)
)
SELECT row_number() OVER() -1 AS pos, sym, finalState
FROM (SELECT unnest(n.tape) AS sym, state AS finalState
   FROM ntape n
   WHERE iter = (SELECT max(iter) FROM ntape)
   ) x;
```

## Part 1.2 Relational Algebra (Total: 29 Points)

### Question 1.2.1 Relational Algebra (2 Points)

Write a relational (bag semantics) algebra expression over the schema from the SQL part (part 1) that returns the names of parties which are *far right*, were established before 1960, and have an endowment that is larger than 2,000,000.

### Solution

$$\pi_{pName}(\sigma_{leaning='farright' \wedge established < 1960 \wedge endowment > 2000000}(\mathbf{party}))$$

### Question 1.2.2 Relational Algebra (4 Points)

Write a relational (bag semantics) algebra expression over the schema from the SQL part (part 1) that returns parties that have gotten more than 3 donations.

### Solution

$$\mathbf{numDon} \leftarrow {}_{pName}\alpha_{count(*) \rightarrow x}(\mathbf{donations})$$
$$\mathbf{q} \leftarrow \pi_{pName}(\sigma_{x>3}(\mathbf{numDon}))$$

### Question 1.2.3 Relational Algebra (4 Points)

Write a relational (bag semantics) algebra expression over the schema from the SQL part (part 1) that returns parties that have no members and have not gotten any donations.

### Solution

$$\pi_{pName}(\textbf{party}) - projection_{pName}(\textbf{donations}) - \pi_{name}(\textbf{member})$$

## Question 1.2.4 SQL → Relational Algebra (5 Points)

Translate the SQL query from Question 1.1.1 into relational algebra (bag semantics).

## Solution

$$\begin{aligned}
\textbf{joins} &\leftarrow \textbf{party} \bowtie_{year \leq oyear \wedge pName=oName} (\delta(\rho_{oName,oyear}(\pi_{pName,year}(\textbf{party})))) \\
\textbf{q} &\leftarrow {}_{sum(amount)}\alpha_{pName,oyear}(\textbf{joins})
\end{aligned}$$

## Question 1.2.5 SQL → Relational Algebra (5 Points)

Translate the SQL query from question 1.1.2 into relational algebra (bag semantics).

## Solution

$$\begin{aligned}
\textbf{totalDon} &\leftarrow {}_{donor,leaning}\alpha_{sum(amount)\to x}(\textbf{donations} \bowtie \textbf{donations}) \\
\textbf{max1} &\leftarrow {}_{leaning}\alpha_{max(x)\to maxAmount}(\textbf{totalDon}) \\
\textbf{donMax1} &\leftarrow \pi_{donor,leaning,maxAmount}(\textbf{max1} \bowtie_{maxAmount=x} \textbf{totalDon} \\
\textbf{max2} &\leftarrow {}_{leaning}\alpha_{max(x)\to maxAmount}(\textbf{totalDon} - \textbf{donMax1}) \\
\textbf{donMax2} &\leftarrow \pi_{donor,leaning,maxAmount}(\textbf{max2} \bowtie_{maxAmount=x} \textbf{totalDon} \\
\textbf{max3} &\leftarrow {}_{leaning}\alpha_{max(x)\to maxAmount}(\textbf{totalDon} - \textbf{donMax1} - \textbf{donMax2}) \\
\textbf{donMax3} &\leftarrow \pi_{donor,leaning,maxAmount}(\textbf{max3} \bowtie_{maxAmount=x} \textbf{totalDon} \\
\textbf{q} &\leftarrow \textbf{donMax1} \cup \textbf{donMax2} \cup \textbf{donMax3}
\end{aligned}$$

## Question 1.2.6 SQL → Relational Algebra (5 Points)

Translate the SQL query from question 1.1.3 into relational algebra (bag semantics).

## Solution

$$\begin{aligned}
\mathbf{posAvg} &\leftarrow {}_{pos,year}\alpha_{avg(salary)\rightarrow x}(\mathbf{positions}) \\
\mathbf{perYearInc} &\leftarrow \rho_{opos,oyear,ox}(\mathbf{posAvg}) \bowtie_{opos=pos \wedge oyear+1=year} \mathbf{posAvg} \\
\mathbf{q} &\leftarrow {}_{year}\alpha_{avg(incr)}(\pi_{x/ox\rightarrow incr}(\mathbf{perYearInc}))
\end{aligned}$$

## Question 1.2.7 Equivalences (4 Points)

Consider the following relation schemas (primary key attributes are underlined):

$R(\underline{A}, B)$, $S(\underline{B}, C)$, $T(\underline{C}, D)$, $U(\underline{E}, F, G)$.

Furthermore, assume that $S.B$ is a foreign key to $R$ and $T.C$ is a foreign key to $S$. Check equivalences that are correct under **bag semantics**. For example $R \bowtie R \equiv R$ should be checked, whereas $R \equiv S$ should not be checked.

- ☐ $R - (S - R) \equiv R - (R - S)$
- ■ $R - (R - S) \equiv S - (S - R)$
- ■ ${}_{G}\alpha_{count(*)}(U) \equiv {}_{G}\alpha_{sum(c)}({}_{F,G}\alpha_{count(*)\rightarrow c}(U))$
- ■ $\pi_C(S \bowtie R) \equiv \pi_C(S)$
- ■ $R \triangleright S \equiv R - (\pi_{A,B}(R \bowtie S))$
- ■ $\delta(\pi_A(R)) \equiv \pi_A(R)$
- ■ $\delta(\pi_A(R \bowtie S)) \equiv \pi_A(R \bowtie S)$
- ☐ $\pi_A(R \bowtie S) \equiv \pi_A(R)$

## Part 1.3 Index Structures (Total: 30 Points)

Assume that you have the following table:

**Item**

| SSN | name | age |
|---|---|---|
| 1 | Pete | 53 |
| 2 | Bob | 57 |
| 44 | John | 20 |
| 43 | Joe | 18 |
| 45 | Alice | 19 |
| 42 | Lily | 99 |
| 88 | Gertrud | 60 |
| 89 | Heinz | 14 |

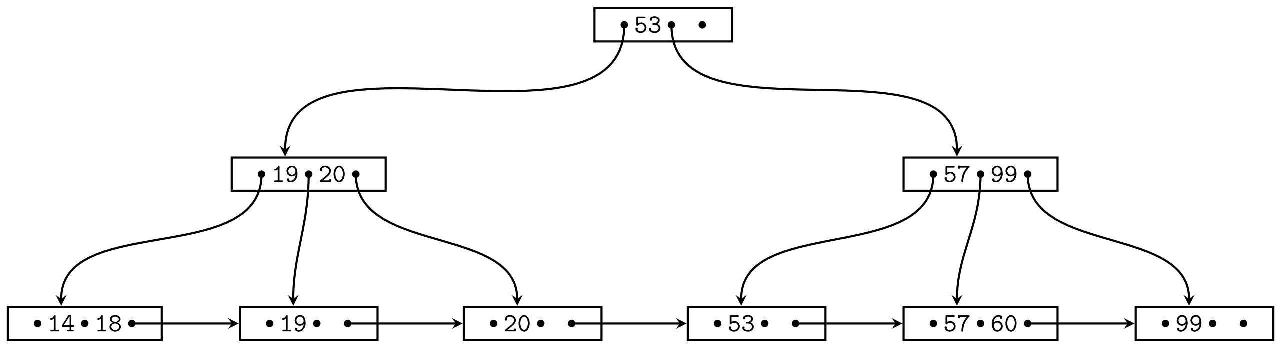
### Question 1.3.1 Construction (12 Points)

Create a B+-tree for table *Item* on key *age* with $n = 2$ (up to two keys per node). You should start with an empty B+-tree and insert the keys in the order shown in the table above. Write down the resulting B+-tree after each step.

When splitting or merging nodes follow these conventions:

- **Leaf Split**: In case a leaf node needs to be split during insertion and $n$ is even, the left node should get the extra key. E.g, if $n = 2$ and we insert a key 4 into a node [1,5], then the resulting nodes should be [1,4] and [5]. For odd values of $n$ we can always evenly split the keys between the two nodes. In both cases the value inserted into the parent is the smallest value of the right node.
- **Non-Leaf Split**: In case a non-leaf node needs to be split and $n$ is odd, we cannot split the node evenly (one of the new nodes will have one more key). In this case the "middle" value inserted into the parent should be taken from the right node. E.g., if $n = 3$ and we have to split a non-leaf node [1,3,4,5], the resulting nodes would be [1,3] and [5]. The value inserted into the parent would be 4.
- **Node Underflow**: In case of a node underflow you should first try to redistribute values from a sibling and only if this fails merge the node with one of its siblings. Both approaches should prefer the left sibling. E.g., if we can borrow values from both the left and right sibling, you should borrow from the left one.

### Solution

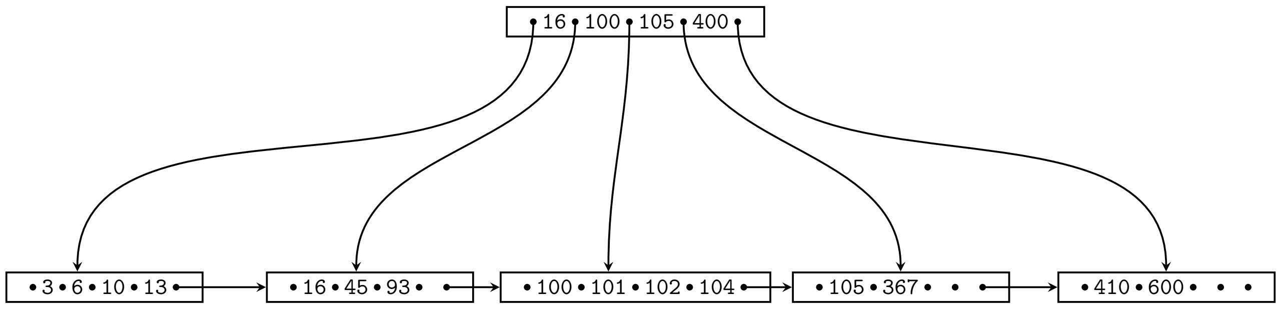


## Question 1.3.2 Operations (10 Points)

Given is the B+-tree shown below ($n = 4$). Execute the following operations and write down the resulting B+-tree after each operation:

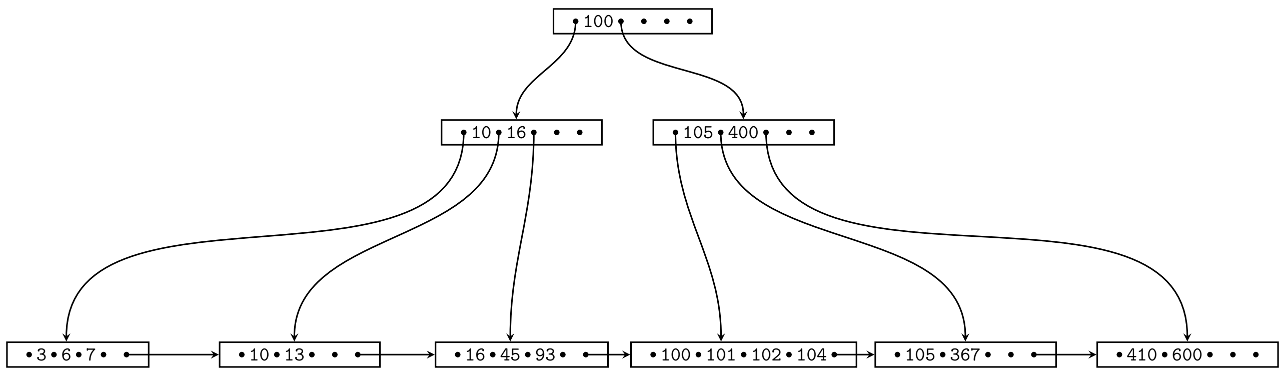
**insert(7), insert(8), insert(103), delete(105), delete(410), insert(1)**

Use the conventions for splitting and merging introduced in the previous question.

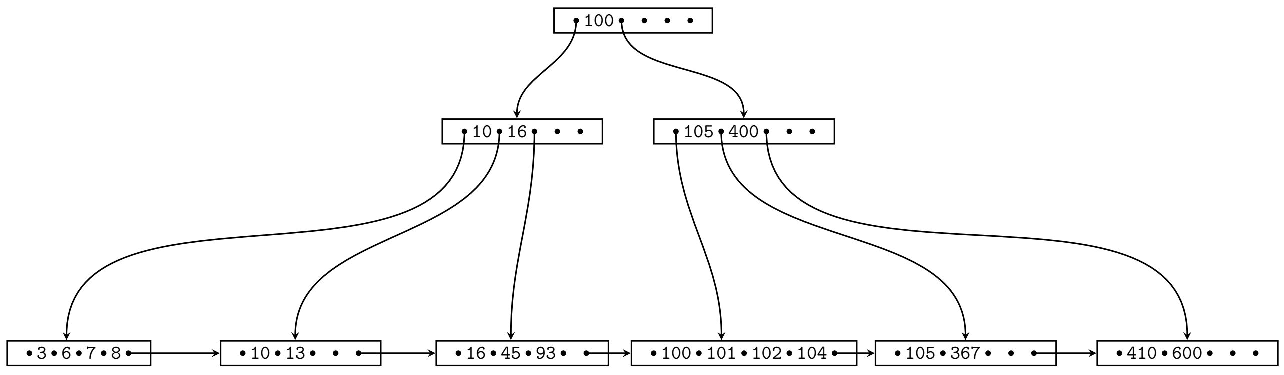


## Solution

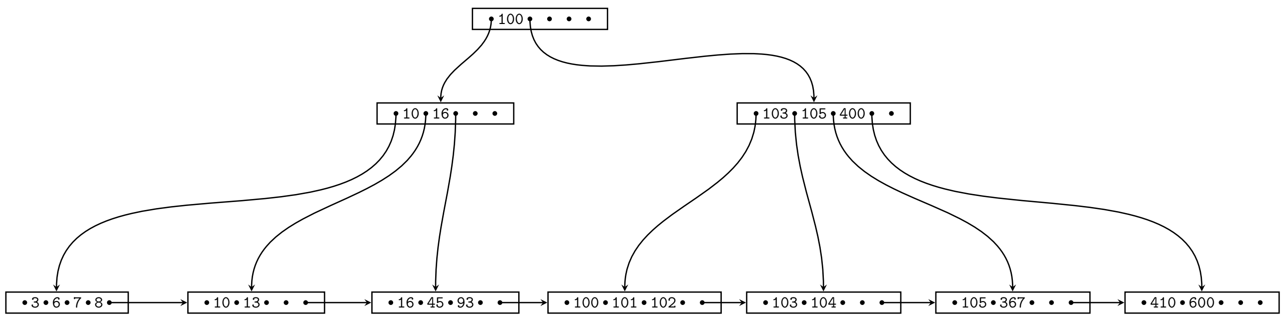
## Insert 7



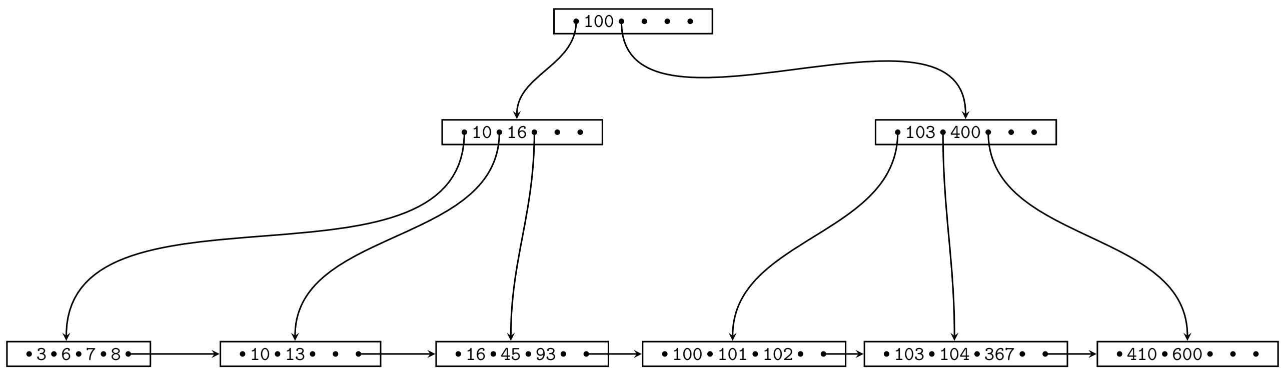
## Insert 8



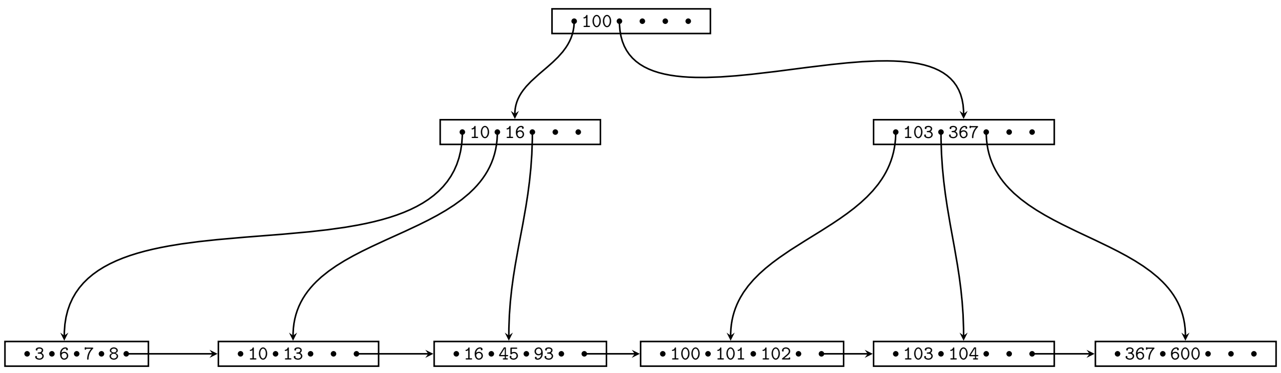
## Insert 103



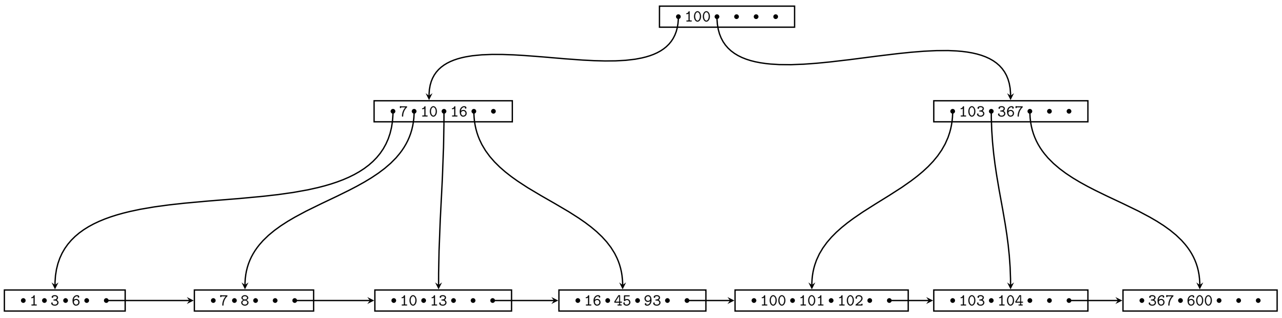
## Delete 105



## Delete 410



## Insert 1



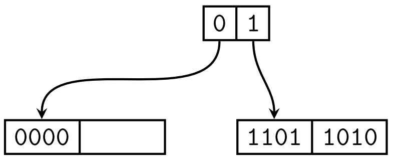
## Question 1.3.3 Extensible Hashing (8 Points)

Consider the extensible Hash index shown below that is the result of inserting values 0, 1, and 2. Each page holds two keys. Execute the following operations

`insert(0),insert(5),insert(8),insert(6)`

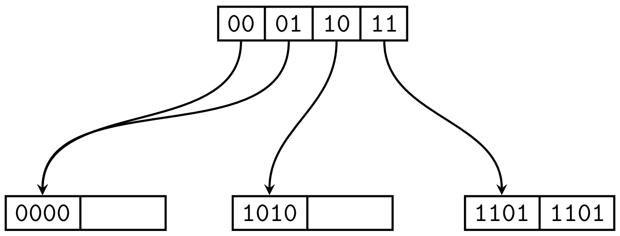
and write down the resulting index after each operation. Assume the hash function is defined as:

| x | h(x) |
|---|---|
| 0 | 1101 |
| 1 | 0000 |
| 2 | 1010 |
| 3 | 1100 |
| 4 | 0001 |
| 5 | 0000 |
| 6 | 1010 |
| 7 | 0111 |
| 8 | 1110 |

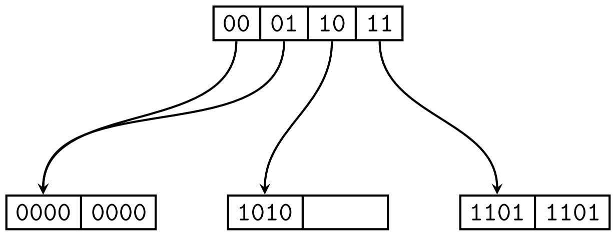


### Solution

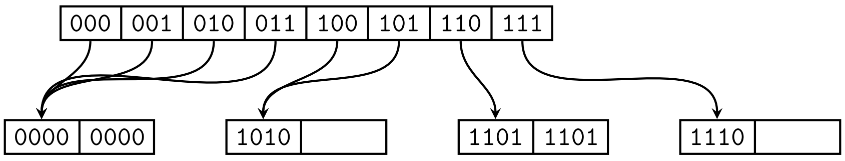
### insert(0)



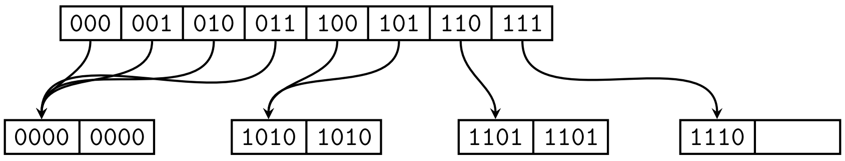
### insert(5)



### insert(8)



### insert(6)



## Part 1.4 Result Size Estimations (Total: 10 Points)

Consider a table `lecture` with attributes `title`, `campus`, `topic`, `roomSize`, a table `student` with `name`, `major`, `age`, and a table `attendsLecture` with attributes `student`, `lecture`, and `hoursAttended`. `attendsLecture.student` is a foreign key to `student`. Attribute `lecture` of relation `attendsService` is a foreign key to of relation `lecture`. Given are the following statistics:

$$
\begin{aligned}
T(lecture) &= 200 & T(student) &= 30,000 & T(attendsLecture) &= 600,000 \\
V(lecture, title) &= 200 & V(student, name) &= 30,000 & V(attendsLecture, student) &= 25,000 \\
V(lecture, campus) &= 3 & V(student, major) &= 10 & V(attendsLecture, lecture) &= 150 \\
V(lecture, topic) &= 10 & V(student, age) &= 30 & V(attendsLecture, hoursAttended) &= 300 \\
V(lecture, roomSize) &= 20
\end{aligned}
$$

### Question 1.4.1 Estimate Result Size (3 Points)

Estimate the number of result tuples for the query $q = \sigma_{topic=DB}(lecture)$ using the first assumption presented in class (values used in queries are uniformly distributed within the active domain).

### Solution

$$T(q) = \frac{T(lecture)}{V(lecture, topic)} = \frac{200}{10} = 20$$

### Question 1.4.2 Estimate Result Size (3 Points)

Estimate the number of result tuples for the query $q = \sigma_{hoursAttended>250}(attendsLecture)$ using the first assumption presented in class. The minimum and maximum values of attribute *hoursAttended* are 0 and 299.

### Solution

$$\begin{aligned} T(q) &= \frac{299-250}{max(hoursAttended) - min(hoursAttended) + 1} \times T(attendsLecture) \\ &= \frac{49}{300} \times 600,000 \sim 97,999.99 \end{aligned}$$

### Question 1.4.3 Estimate Result Size (4 Points)

Estimate the number of result tuples for the query $q$ below using the first assumption presented in class.

$$q = (student \bowtie_{name=student} \sigma_{hoursAttended \neq 0}(attendsLecture) \bowtie_{lecture=title} \sigma_{campus=Mies}(lecture)$$

### Solution

$$\begin{aligned}
q_1 &= \sigma_{hoursAttended \neq 0}(attendsLecture) \\
T(q_1) &= = \frac{T(attendsLecture) \times (V(attendsLecture, hoursAttended) - 1}{V(attendsLecture, hoursAttended)} = \frac{600,000 \times 299}{300} = 598,000 \\
V(q_1, student) &= V(attendsLecture, student) = 25,000
\end{aligned}$$

$$V(q_1, lecture) = V(attendsLecture, lecture) = 150$$

$$\begin{aligned}
q_2 &= \sigma_{campus=Mies}(lecture) \\
T(q_2) &= = \frac{T(lecture)}{V(lecture, campus)} = \frac{200}{3} \sim 66.67 \\
V(q_2, title) &= min(V(lecture, title), T(q_2)) = 66.67
\end{aligned}$$

$$\begin{aligned}
T(q) &= \frac{T(student) \times T(q_1) \times T(q_2)}{max(V(student, name), V(q_1, student)) \times max(V(q_1, lecture), V(q_2, title))} \\
&= \frac{30,000 \times 598,000 \times 66.67}{max(30,000, 25,000) \times max(150, 66.67)} \sim 265,791
\end{aligned}$$