Types, Expressions, and States

CS 536: Science of Programming, Spring 2023

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A. Why

- Expressions represent values relative to a state.
- Types describe common properties of sets of values.
- The value of an array is a function value from index values to array values.

B. Outcomes

At the end of today, you should

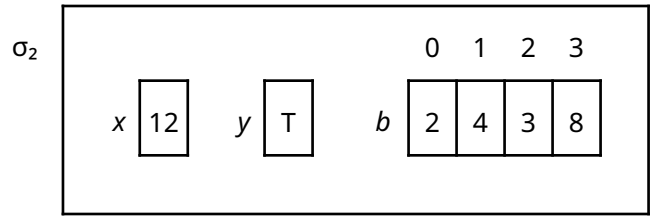
- Be able to read and write expressions we'll be using in our language.
- Be able to read and write states.
- Be able to evaluate an expression relative to a state.
- Be able to handle array names, array indexing expressions, and their values relative to a state.

C. Questions

1. Which of the following expressions are legal or illegal according to the syntax we're using? Assume *x*, *y*, *z* are integer variables and *b* is an array name.
   a. ***if*** *x* > *y* ***then*** *x* ***else*** *y* ***fi*** -- Is it important that *x* and *y* have the same type?
   b. ***if*** *x* < *y* ***then*** *-1* ***else if*** *x* = *y* ***then*** *0* ***else*** *1* ***fi fi***
   c. ***if*** *y* = *0* ***then*** *f* ***else*** *g* ***fi*** (*17*) -- Assume *f* and *g* are functions
   d. *b[0][1]* -- Assume *b* is 2-dimensional array
   e. *b* -- Assume *b* is an array
   f. *f*(*b*, *b[0]*) < *3* -- What type for *f* makes this legal?
   g. ***if*** *x* < *3* ***then*** *x* ***else*** *F* ***fi***

2. Which of the following are legal ways to write out a state? (And if not, why not?)
   a. {*x* = 5, *y* = 2}
   b. {*x* = five, *y* = one plus one}
   c. {*x* = 5, *y* = *x* minus 3}
   d. {*x* = α, *y* = α - 3} where α = 5
   e. {*x* = 5, *y* = (the value of *x* in this environment minus 3)}
   f. { }
   g. σ = {*x* = 5, *y* = σ(*x*) minus 3} (How is this different from (c) above?)

3. Consider the state σ_2 described graphically below.



   a. Write a definition for $\sigma_2 = \{ \ldots \}$ using four ways described in the class 3 notes (specifically, ***Arrays and Their Values***).
   b. Calculate $\sigma_2(e)$ where $e \equiv y \wedge x > b[x/5]$. Remember, integer division truncates: $\varnothing(5/3) = \underline{1}$

4. Let $\sigma_3 \equiv \{ z = 4, b[0] = 1, b[1] = 5, b[2] = 8 \}$.
   a. Abbreviate this using tuple notation for the value of b (i.e., $b = (\ldots)$).
   b. Write out the graphical representation of σ_3 (a memory diagram as in Problem 2).
   c. Calculate $\sigma_3(e)$ where $e \equiv$ *if b[b[z-4]] > z then z+2 else z-2* ***fi*** (Hint: Give names to parts of e and calculate the values of those parts first.)

5. Let $e_4 \equiv x = y+1 \wedge y = z^2 - 3 \wedge z = 6$. Write out the textual definition of a state σ_4 in which e_4 evaluates to true. Use only bindings that map variables to constants. $\sigma_4 = \{ x = 34, y = 33, z = 6 \}$

6. Which of the following states are well-formed and also proper for the expression *b[i] + 0 * y*? If ill-formed, why? If taking the value might cause a runtime error, why?
   a. $\{i = 0, b = (3, 4, 8), y = 3, z = 5)$
   b. $\{i = 0, b = (6), y = 5)$
   c. $\{i = 0, b = 6, y = 5)$
   d. $\{i = 1, b = (3, 4, 8))$
   e. $\{i = 1, i = 2, y = 0, b = (2, 6))\}$
   f. $\{i = 5, b = (1, 2), y = 4\}$

CS 536 Solution to Practice 3 (Types, Expressions, and States)

1. (Legal and illegal expressions)
   a. ***if*** *x* > *y* ***then*** *x* ***else*** *y* ***fi*** is legal
   b. ***if*** *x* < *y* ***then*** *-1* ***else if*** *x* = *y* ***then*** *0* ***else*** *1* ***fi fi*** is legal
   c. ***if*** *y* = *0* ***then*** *f* ***else*** *g* ***fi*** (*17*) is illegal because the conditional expression can't yield a function
   d. *b[0][1]* is legal (*b* must be a 2-dimensional array)
   e. *b* (all by itself) is illegal, since *b* we've assumed is an array
   f. *f*(*b*, *b[0]*) < *3* is legal (the name *b* is being used as an argument to a function). We infer that *f* has type (int array) × int → int.
   g. ***if*** *x* < *3* ***then*** *x* ***else*** *F* ***fi*** is illegal because *x* and *F* have different types. (I.e., the expression doesn't have a fixed type because the types of its arms don't match.)

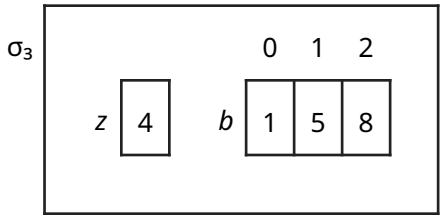
2. (Legal ways to represent states)
   a. {*x* = 5, *y* = 2} is legal
   b. {*x* = five, *y* = one plus one} is legal because "five" and "one" etc. refer to semantic objects.
   c. {*x* = 5, *y* = *x* minus 3} is illegal: *x* = 5 tells us *x* is a syntactic variable, but in *y* = *x* minus 3, *x* has to be a semantic value because the binding is *y* = some value. This is inconsistent. Said another way, from *x* = 5, we know *x* is a variable that could appear in a program. If we want to say "the value of *y* is the value of x, minus 3" Then (g) below is the way to do it.
   d. {*x* = α, *y* = α - 3} where α = 5 — is legal. We infer that symbols *x* and *y* are syntactic objects and α names a semantic object, 5.
   e. {*x* = 5, *y* = (the value of *x* in this environment, minus 3)} is legal. Since "the value of *x* in this environment" is just another name (albeit complicated) for the mathematical object 5, it's legal to use here.
   f. { } is legal, since it's just another way to write ∅, the empty state.
   g. σ = {*x* = 5, *y* = σ(x) *minus* 3} is legal — it's the way (c) could be rewritten to be legal.

3. (Graphically defined state)
   a. σ_2 = {b = β, *x* = 12, *y* = T} where where β = (2, 4, 3, 8).
      σ_2 = {*b[0]* = 2, *b[1]* = 4, *b[2]* = 3, *b[3]* = 8, *x* = 12, *y* = T}.
      σ_2 = {b = β, *x* = 12, *y* = T} where β = {(0, 2), (1, 4), (2, 3), (3, 8)}.
      σ_2 = {b = β, *x* = 12, *y* = T} where β(0) = 2, β(1) = 4, β(2) = 3, β(3) = 8.

   b. You can write out these kinds of calculations to different levels of detail, but a brief answer is that $\sigma_2(e) = \sigma_2(y \wedge x > b[x/5]) = T \wedge 12 > 3 = T$ You can certainly show intermediate steps:
      $$\sigma_2(e) = \sigma_2(y) \wedge \sigma_2(x) > \sigma_2(b)(\sigma_2(x/5))$$

$= T \wedge 12 > \sigma_2(b)(12/5)$
$= T \wedge 12 > \sigma_2(b)(2) = T \wedge 12 > 3 = T$

4. (Alternative ways to represent a state with an array value)
   a. $\sigma_3 = \{z = 4, b = (1, 5. 8)\}$
   b.



   c. We have $e \equiv$ ***if*** *b[b[z-4]] > z* ***then*** *z+2* ***else*** *z-2*. To make this easier to deal with, let's break it down. Let $e \equiv$ ***if*** e_1 > *z* ***then*** *z+2* ***else*** *z-2* where $e_1 \equiv b[e_2]$ and $e_2 \equiv b[z-4]$.
      - First, $\sigma_3(e_2) = \sigma_3(b[z-4]) = (\sigma_3(b))(\sigma_3(z-4)) = (\sigma_3(b))(4–4) = (\sigma_3(b))(0) = 1$
      - So $\sigma_3(e_1) = \sigma_3(b[e_2]) = (\sigma_3(b))(\sigma_3(e_2)) = (\sigma_3(b))(1) = 5$
      - Then $\sigma_3(e_1 > z) = (\sigma_3(e_1) > \sigma_3(z)) = 5 > 4 = F$
      - So $\sigma_3(e) = \sigma_3($ ***if*** $e_1 > z$ ***then*** *z+2* ***else*** *z-2*$) = \sigma_3(z+2)$ because the test $\sigma_3(e_1 > z) = F$
      - So finally, $\sigma_3(e) = \sigma_3(z+2) = \sigma_3(z) + \sigma_3(2) = 4+2 = 6$

5. $\sigma_4 = \{z = 6, y = 33, x = 34\}$

6. (Proper states)
   a. (Well-formed and) Proper: The extra binding for z isn't a problem
   b. (Well-formed and) Proper: The value of *b* is an array of length 0.
   c. (Well-formed but) Improper: The value of *b* can't be an integer.
   d. (Well-formed but) Improper: We need a binding for *y* even though we're multiplying it by zero. [So our semantics uses eager evaluation, not lazy evaluation.]
   e. Ill-formed: We have two bindings for i.
   f. (Well-formed and) Proper but causes a runtime error, since *b* has size 2.

