**Weakest Preconditions**

**Part 1: Definitions and Basic Properties**

*CS 536: Science of Programming, Fall 2021*

A. Why

- Weakest liberal preconditions \((w/p)\) and weakest preconditions \((wp)\) are the most general requirements that a program must meet to be correct.

B. Objectives

At the end of this activity you should be able to

- Define what a weakest liberal precondition \((wlp)\) and weakest precondition \((wp)\) is and how it's related to (and different from) preconditions in general
- Be able to calculate the \(wlp\) of a simple loop-free program.

C. Problems

1. Let \(w \leftrightarrow w p(S, q)\), let \(S\) be deterministic, and let \(\{t\} = M(S, \sigma)\) where \(t \in \Sigma \cup \{\bot\}\).
   a. For which \(\sigma \models w\) do we have \(\sigma \models \text{tot} \{w\} S \{q\}\)?
   b. For which \(\sigma \models \neg w\) do we have \(\sigma \models \text{tot} \{\neg w\} S \{q\}\)? How about \(\sigma \models \{\neg w\} S \{q\}\)?
   c. For which \(\sigma \models w\) do we have \(\sigma \models \text{tot} \{w\} S \{\neg q\}\)?
   d. For which \(\sigma \models \neg w\) do we have \(\sigma \models \{\neg w\} S \{\neg q\}\)?
   e. If \(S\) is nondeterministic, how do we have to modify the statement in part (d)?

2. If \(\sigma \models w\) and \(\sigma \models \{w\} S \{q\}\) and \(\sigma \not\models \text{tot} \{w\} S \{q\}\),
   a. What can we conclude about \(M(S, \sigma)\)?
   b. If in addition, \(S\) is deterministic, what more can we conclude about \(M(S, \sigma)\)?

3. For an arbitrary \(p\) (not necessarily one that implies \(w\)), what \(\models\) and \(\models \text{tot}\) properties relationships do the triples
   a. \(\{p \land w\} S \{q\}\) and \(\{\neg p \land w\} S \{q\}\) have?
   b. \(\{p \land \neg w\} S \{\neg q\}\) and \(\{\neg p \land \neg w\} S \{\neg q\}\) have, if \(S\) is deterministic?
   c. \(\{p \land \neg w\} S \{q\}\) and \(\{\neg p \land \neg w\} S \{q\}\) have, if \(S\) is nondeterministic?
4. How are $wp(S, q_1 \lor q_2)$ and $wp(S, q_1) \cup wp(S, q_2)$ related if $S$ is deterministic? If $S$ is nondeterministic?

5. Briefly explain why each of the following statements about $wp$ and $wlp$ are correct. (Answers like “That's how $X$ is defined” are allowed.)
   a. For all $\sigma \in \Sigma$, $\sigma \models wp(S, q) \iff M(S, \sigma) = q$
   b. For all $\sigma \in \Sigma$, $\sigma \models wlp(S, q) \iff M(S, \sigma) - \bot \models q$
   c. $\models_{tot} \{wp(S, q)\} S \{q\}$
   d. $\models \{wlp(S, q)\} S \{q\}$
   e. $\models_{tot} \{p\} S \{q\} \iff p \rightarrow wp(S, q)$
   f. $\models \{p\} S \{q\} \iff p \rightarrow wlp(S, q)$
   g. $\models \{-wp(S, q)\} S \{-q\}$, if $S$ is deterministic
   h. $\models_{tot} \{-wlp(S, q)\} S \{-q\}$, if $S$ is deterministic
   i. $\nvdash p \rightarrow wp(S, q) \iff \nvdash_{tot} \{p\} S \{q\}$
   j. $\nvdash p \rightarrow wlp(S, q) \iff \nvdash \{p\} S \{q\}$

6. Which of the following statements about relationships between $wp$ and $wlp$ are possible and which are impossible? Briefly explain why or why not.
   a. $wlp(S, q) \land wlp(S, \neg q)$
   b. $\neg wp(S, q) \land \neg wp(S, \neg q)$
   c. $wp(S, q) \land \neg wlp(S, q)$
   d. $wlp(S, q) \land \neg wp(S, \neg q)$
   e. $wp(S, q) \land \neg wlp(S, \neg q)$
   f. For deterministic $S$, $\neg wp(S, q) \land \neg wp(S, \neg q)$ and $M(S, \sigma) - \bot \neq \emptyset$
   g. For deterministic $S$, $\neg wp(S, q) \land \neg wp(S, \neg q)$ and $\bot \notin M(S, \sigma)$