Syntactic Substitution, Forward Assignment, & sp

CS 536: Science of Programming, Fall 2022

Due Wed Mar 8, 11:59 pm

2023-03-02 p.1

Problems [60 points total]

Class 12: Syntactic Substitutions [30 points]

For Problems 1 – 4, Let $p \equiv x - y < f(a) \lor \forall x. x \ge a * y \rightarrow \exists y. f(x - y) > a + y * z$ and calculate the substitutions below. Show some detail if you want partial credit for a wrong answer. Just do the syntactic calculations. Don't do any arithmetic or logical simplifications.

- 1. [4 points] p[y+z/x]
- 2. [6 points] p[a-y/y]
- 3. [9 points] p[a*y/a]
- 4. [11 points] $p[x \div y / a][y z / x]$

Lecture 13: Forward Assignment; Strongest Postconditions [30 points]

- 5. [3 points] Give an example of an S such that \models {*T*} S {*sp*(*T*, S)} but \nvDash_{tot} {*T*} S {*sp*(*T*, S)}.
- 6. [5 points] Syntactically calculate $sp(x < y \land x + y \le n, x := f(x + y); y := g(x * y))$. Don't simplify the result.

For Problems 7 – 9, calculate each *sp* or *wp* result syntactically. If simplification is requested, do the syntactic calculation first, then simplify, maintaining logical equivalence unless asked otherwise.

- 7. [5 points] Calculate and then logically simplify sp(x < x * k, x := x/2).
- 8. [5 points] Calculate (but don't simplify) wp(x := x/2, x < x * k).
- 9. [12=2*6 points]. Let $S \equiv if even(n)$ then n := n-1 fi. (Let $even(x) \equiv x \% 2 = 0$ and $odd(x) \equiv x \% 2 = 1$. You can treat $\neg even(x) \equiv odd(x)$ if you like.)
 - a. Calculate and then logically simplify *wp(S, odd(n))*. [2023-03-02]
 - b. Calculate and then logically simplify $sp(n = n_0, S)$. As part of simplification, drop n_0 . (The simplified result will be strictly weaker than the unsimplified result.)