Weakest Preconditions 1 & 2; Domain Predicates

CS 536: Science of Programming, Spring 2023

Due Mon Feb 27, 11:59 pm

Problems [60 points total]

Class 10: Weakest Preconditions part 1 [27 points]

1. [*3* points] Let $IF \equiv$ *if* $B_1 \rightarrow S_1 \ \square \ B_2 \rightarrow S_2$ *fi* and let $w_1 \Leftrightarrow wlp(S_1, q)$ and $w_2 \Leftrightarrow wlp(S_2, q)$. Question: Why is $wlp(IF, q) \Leftrightarrow (B_1 \rightarrow w_1) \wedge (B_2 \rightarrow w_2)$ but not $(B_1 \wedge w_1) \vee (B_2 \wedge w_2)$.

2. [*4* points] Which of the following statements behave differently depending on whether S is deterministic or nondeterministic? Explain briefly.
   - $wp(S, p \vee q) \rightarrow wp(S, p) \vee wp(S, q)$
   - $wp(S, p) \vee wp(S, q) \rightarrow wp(S, p \vee q)$
   - $wp(S, p \wedge q) \rightarrow wp(S, p) \wedge wp(S, q)$
   - $wp(S, p) \wedge wp(S, q) \rightarrow wp(S, p \wedge q)$

3. [*20 = 5 *4* points] Consider the statement $\sigma \models \{wlp(S, q)\} S \{q\}$. If σ satisfies the precondition, then $\sigma \models \{wlp(S, q)\} S \{q\}$ is satisfied when
   - $M(S, \sigma) - \perp \models q$ (description using meaning functions)
   - For all $\tau \in M(S, \sigma)$, $\tau = \perp$ or $\tau \models q$ (equivalent description using states/logic)

   (Note the description is correct whether S is deterministic or nondeterministic.)

   For each of the statements below, assume σ satisfies the precondition and give the meaning function requirement and the equivalent logical description. If not specified, S could be deterministic or nondeterministic. If S is deterministic, it could be helpful to use the phrase $M(S, \sigma) = \{\tau\}$.

   a. $\sigma \not\models \{wlp(S, q)\} S \{q\}$
   b. $\sigma \models_{\text{tot}} \{\neg wlp(S, q)\} S \{\neg q\}$, if S is deterministic
   c. $\sigma \models_{\text{tot}} \{wp(S, q)\} S \{q\}$
   d. $\sigma \not\models_{\text{tot}} \{wp(S, q)\} S \{q\}$
   e. $\sigma \models \{\neg wp(S, q)\} S \{\neg q\}$, if S is deterministic

Class 11: Weakest Preconditions part 2 [9 points]

4. [*9* points] Calculate $wlp\,(\,\textbf{\textit{if}}\ x < 0\ \ \textbf{\textit{then}}\ x := -x\ \ \textbf{\textit{fi}}, x^2 \geq x\,)$. (Don't forget the implicit "***else skip***" clause.) You can omit intermediate calculations but they might be worth partial credit. After syntactically calculating the *wlp*, logically simplify the result. (Textual simplifications like $p \wedge p \equiv p$ are always allowed.)
   a. [*3* points] Calculate the *wlp* of the true branch
   b. [*2* points] Calculate the *wlp* of the false branch
   c. [*2* points] Calculate the overall *wlp*.
   d. [*2* points] Give the result after logical/arithmetic simplification.

Class 11: Domain Predicates [24 points]

Calculate the *wp* 's below. Show your intermediate calculations. You can logically simplify your answer as you go and/or at the end or not at all (your preference). (Textual simplifications like $p \wedge p \equiv p$ are always allowed.)

5. [*12 = 4 *3* points] $wp\,(S, q)$ where $S \equiv\ \ y := y/x$ and $q \equiv sqrt\,(y) < x$.
   a. Calculate $D\,(S)$.
   b. Calculate $w \equiv wlp\,(S, q)$.
   c. Calculate $D\,(w)$.
   d. Calculate $wp\,(S, q)$ (it's $\Leftrightarrow D\,(S) \wedge w \wedge D\,(w)$).

6. [*12 = 4 *3* points] $wp\,(S, q)$ where $S \equiv \textbf{\textit{if}}\ y \geq 0\ \ \textbf{\textit{then}}\ x := y/x\ \ \textbf{\textit{else}}\ x := -x/y\ \ \textbf{\textit{fi}}$ and $q \equiv r < x \leq y$.
   a. Calculate $D\,(S)$.
   b. Calculate $w \equiv wlp\,(S, q)$.
   c. Calculate $D\,(w)$.
   d. Calculate $wp\,(S, q)$.

