Sequential Nondeterminism, Hoare Triples 1 & 2

CS 536: Science of Programming, Spring 2023

Due Thu Feb 16, 11:59 pm [not Sep 16]

2023-02-13: p.1; 2023-02-15: p.2

A. Problems [60 points total]

Class 7: Sequential Nondeterminism

1. [$12 = 2 * 6$ points] Let DO be the nondeterministic loop [2023-02-13 do/od keywords]

   $$\textbf{do}\ x \neq 0 \rightarrow x := x - 1;\ y := y + 1 \ \square\ x \neq 0 \rightarrow x := x - 1;\ y := y + 2\ \textbf{od}$$

   a. First, let's work on what what a typical loop iteration does over an arbitrary state $\sigma = \{x = \beta, y = \delta\}$. Assume $\beta \geq 2$ and calculate the two states we can be in after a single iteration of the loop. I.e., what are the τ where $\langle DO, \sigma\rangle \rightarrow^3 \langle DO, \tau\rangle$?

   b. Extend part (a) to do κ iterations where $1 < \kappa \leq \beta$. What is the set of final states Σ' we can reach in 3κ iterations? I.e., what is $\Sigma' = \{\tau \in \Sigma \mid \langle DO, \sigma\rangle \rightarrow^{3\kappa} \langle DO, \tau\rangle\}$?

Classes 8 & 9: Hoare Triples, pt 1 & 2

2. [$16 = 4 * 4$ points]

   a. Using backward assignment, what can we use for precondition p_1 in the triple $\{p_1\}\ b := b + b\ \{b * c \leq d - b\}$? (Mild hint: Be careful with parenthesization)

   b. Using backward assignment, what can we use for p_2 in $\{p_2\}\ x := m\ \{1 \leq x * y \leq n * m\}$?

   c. Using backward assignment, what can we use for p_3 in $\{p_3\}\ y := n\ \{p_2\}$?

   d. Joining parts (b) and (c), what can we use for p_4 in $\{p_4\}\ y := n;\ x := m\ \{1 \leq x * y \leq n * m\}$?

3. [$6 = 2 * 3$ points] Let $p_0 \rightarrow p$, $p \rightarrow p_1$, $q_0 \rightarrow q$, and $q \rightarrow q_1$ all be valid. From $\{p\}\ S\ \{q\}$, there are four triples of the form $\{p_i\}\ S\ \{q_j\}$ that get by replacing p by p_0 or p_1 and q by q_0 or q_1.

   a. If $\sigma \models \{p\}\ S\ \{q\}$, which of the four triples $\sigma \models \{p_i\}\ S\ \{q_j\}$ is/are also satisfied by σ under $\models$? Briefly justify.

   b. Repeat part (a) but under total correctness.

4. [$8 = 2 * 4$ points] Say $\sigma \models \{p_1\}\ S\ \{q_1\}$ and $\sigma \models \{p_2\}\ S\ \{q_2\}$.

   a. Does $\sigma \models \{p_1 \wedge p_2\}\ S\ \{q_1 \vee q_2\}$? Justify briefly.

   b. Does $\sigma \models \{p_1 \vee p_2\}\ S\ \{q_1 \wedge q_2\}$? Justify briefly.

5. [*10* points] Answer the following questions below about the relationships between or variations of correctness triples. Assume $\sigma \neq \bot$ and S is deterministic.

For (a) and (b), the four statements are $\sigma \models \{p\}S\{q\}$, $\sigma \models \{p\}S\{\neg q\}$, $\sigma \not\models \{p\}S\{q\}$, $\sigma \not\models \{p\}S\{\neg q\}$

   a. [*4* points] There are four statements of the form $\sigma\ (\models \text{ or } \not\models)\ \{p\}\ S\ \{q \text{ or } \neg q\}$. Which (if any) of them are implied by $\sigma \models_{tot} \{p\}\ S\ \{q\}$?

   b. [*4* points] There are [2023-02-15] four statements of the form $\sigma\ (\models \text{ or } \not\models)\ \{p\}\ S\ \{q \text{ or } \neg q\}$. Which (if any) of them are implied by $\sigma \models_{tot} \{T\}\ S\ \{q\}$?

For (a) and (c), the four statements are $\sigma \models \{p\}S\{q\}$, $\sigma \models \{p\}S\{\neg q\}$, $\sigma \models \{\neg p\}S\{q\}$, $\sigma \models \{\neg p\}S\{\neg q\}$

   c. [*2* points] There are four statements of the form $\sigma \models \{p \text{ or } \neg p\}\ S\ \{q \text{ or } \neg q\}$. When can all four of them be satisfied at the same time, or is it impossible?

Definitions

"Σ_0 ***partly*** $\models p$" means there is a $\tau \in \Sigma_0$ with $\tau \models p$.

"Σ_0 ***partly*** $\not\models p$" means there is a $\tau \in \Sigma_0$ with $\tau \models \neg p$.

6. [$8 = 2 * 4$ points] Now assume that $\sigma \neq \bot$ and S is nondeterministic and answer the following questions.

   a. There are four statements of the form σ partly ($\models$ or $\not\models$) $\{p\}\ S\ \{q \text{ or } \neg q\}$. If $\bot \notin M(S, \sigma)$, then which (if any) of them are implied by $\sigma \not\models \{p\}\ S\ \{q\}$?

   b. Continuing, which (if any) of the remaining statements can occur (but might not) when $\sigma \not\models \{p\}\ S\ \{q\}$?