Types, Expressions, States, Quantified Predicates

CS 536: Science of Programming, Spring 2023

Due Thu Feb 2, 11:59 pm

2023-02-01: p.2

A. Formatting and Submitting Your Work

- Remember to use a word processor to write out your answers. Quantified variables range over $\mathbb{Z}$ unless otherwise specified.
- To speed up grading, be sure to include your group number and a list of the group members' userids and A# numbers. Use *.pdfs, not *.doc files.
- In all the problems below, assume that variables written with different names are $\neq$ unless explicitly said otherwise. (So with x and y, assume $x \neq y$, but if we define "$v \equiv x$ or y", we don't necessarily know if $v \equiv x$ or $v \neq x$.)

B. Problems [60 points total]

Class 3: Types, Expressions, and Arrays

1. (*4* points) Let b be a two-dimensional array. Is the expression $b[0] + b[2][3]$ legal or illegal according to the syntax we're using. If illegal, why? If legal, what is the type of the resulting expression?

2. (*4* points) Is $\{u = (4), w = u[1], r = one, s = four, t = r + s\}$ a well-formed state? If not, why?

3. (*6 = 2 * 3* points) Let $\sigma = \{x = 2, b = \beta\}$ where $\beta = (7, 12, 3)$. Complete these equivalent definitions of σ:
   a. $\sigma = \{x = 2, b = \{(0, 7), \ ...$
   b. $\sigma = \{x = 2, b[0] = 7, ...$

4. (*8 = 2 * 4* points) Take the expression $c * b[b[k]]$. For each state below, is it well-formed; if it is, is it also proper for the expression? If it is, does the expression terminate correctly (and with what result)? If not, why?
   a. $\{k = 0, \ b = \underline{(3, 6, 1, 4)}, c = \underline{2}\}$
   b. $\{k = \underline{0}, \ b = \underline{3}\}$

Class 4: State Updates, Satisfaction of Quantified Predicates

5. (*8 = 2 * 4* points) Let $\sigma = \{ x = \underline{2}, y = \underline{4} \}$.
   a. What is $\sigma[x \mapsto \underline{1}]$; what is $\sigma \cup \{(x, \underline{1})\}$; are they equal?
   b. What is $\sigma[v \mapsto \underline{2}]$; what is $\sigma \cup \{(v, \underline{2})\}$; are they equal?

6. (*8 = 2 * 4* points) Let $\sigma = \{ x = \underline{2}, y = \underline{4} \}$. (Assume $x \neq y$.)
   a. What is $\sigma[x \mapsto \sigma(y)][y \mapsto \sigma(x)]$? (Be careful)
   b. Let $\tau = \sigma[x \mapsto 3][y \mapsto \tau(x) * 4]$. What is τ? (Be careful)

7. (*8 = 2 * 4* points) For each of the following, say whether the state satisfies the quantified predicate (and if not, briefly why). Give a witness value (for satisfied existentials) or a counterexample (for unsatisfied universals).
   a. Does $\{ x = \underline{0}, b = \underline{(5, 3, 6)} \} \models (\forall x . \forall k . 0 < k < 3 \wedge x < b[k])$?
   b. Does $\{ b = (2, 5, 4, 8) \} \models (\exists m . 0 \leq m < 4 \rightarrow b[m] < 2)$?

8. (*4* points) Explain (in words): When does the following property hold?
   $\not\models (\exists x \in V . (\exists y \in U . P(x, y)) \wedge (\forall z \in W . Q(x, z)))$

9. (10 points) Write a definition for a predicate function *Unique*(b, x, m) that yields true when the m array elements starting with $b[x]$ have unique values (i.e., m array locations, m different values). For example, *Unique*$((1, 2, 3, 2), 1, 2)$ is true but [2023-02-01] *Unique*$((1, 2, 3, 2), 1, 3)$ is false because it contains *2* twice. If $m < 0$ or any of the indexes x, $x + 1$, etc. are illegal, return false. If you like, you can assume that $\wedge$ and $\vee$ are short-circuiting, like && and || in C and Java. Feel free to define helper predicates if you want.