CS 536 Class 24
Thu 4/22

We have disjoint programs $k \neq j$

pairwise change $(S_k)$ cause $(S_j) = \emptyset$

No memory changing done by
1 thread to another

If $[S_i \cup S_j \cup S_n]$ is DPP

then $\{p^3 S_i\} - [S_n \{q^3\}$

implies $\{p^3 \{S_i \cup S_j \cup S_n\} \{q^3\}$

sequentialization

DPPs have a unique result (execution order doesn't matter)

Seg's rule kind of partial

Need lots of intermediate conditions, carried across seq.
Would really like to reason about threads more independently and combine their pre/post-conditions

\[ \{ p_1 \} s_1[18] + \{ p_2 \} s_2[90] \]

and be able to say \[ \{ p_1 \land p_2 \} s_1[5, 11 \land s_2] \]

parallelism rule

You'd think that DPP's would be ok here.

\[ S_1 \quad \{ x=0 \} x_1 = 1 \quad \{ x = 13 \} x_2 = 0 \quad \{ x=9 \} \]

\[ S_2 \quad \{ x=3 \} \{ x = 1 \land y = 0 \} x_1 = 4 \quad \{ x = 1 \land x = y \} \]

false!

Why? bec. \( S_2 \) is changing \( x \) + \( S_2 \)'s condition

Also need change \( (S_1) \land \text{Free}(\text{condition of } S_2) \)

\[ \{ x \} \land \{ x, y \} = \{ x \} \neq \emptyset \]

Even though change \( (S_1) \land \text{vars}(S_2) = \emptyset \), we still get
Free (condition) = free vars of condition

\[ S_1 \cdot \{ p \} S_2 \{ q \} \]  
\[ S_1 \text{ is disjoint from } \text{conditions of } \{ p \} S_2 \{ q \} \]

if \( \text{change}(S_1) \cap \text{free}(p \cup q) = \emptyset \)

also

\[ S_2 \text{ disjoint from } \text{conditions of } \{ p \} S_2 \{ q \} \]

\[ \text{change}(S_2) \cap \text{free}(p \cup q) = \emptyset \]

To get a parallelism rule,

we need DRPs with pairwise disjoint conditions

\[ \text{change}(S_j) \cup \text{use}(S_j) = \emptyset \]
\[ \cup \text{free}(S_j^*) = \emptyset \]

\( S^* \) is conditions + \( S_j \) triple

If these hold, then parallelism is ok

\( \{ p, S_1 \{ q \} \}, \{ p_2 \} S_2 \{ q_2 \} \)

and they are disjoint parallel

\( \{ p_2 \} S_2 \{ q_2 \} \] and disjoint conditions
Examples

\begin{align*}
S_1: & \begin{cases}
  y \geq 0 \\
  x = y - \{ y = 0 \} x = y^3
\end{cases} \\
& \begin{cases}
  x \geq 3 \\
  x = x + 1 \{ x \geq 3 \}
\end{cases} \\
S_1 + S_2 & \text{disj}
\end{align*}

\begin{align*}
S_2: & \begin{cases}
  y \geq x \}
  y = y + 1 \{ y \geq x \}
\end{cases} \\
& \begin{cases}
  x \geq y
  y = y + 1 \{ x \geq y \}
\end{cases}
\end{align*}

\begin{align*}
\text{disj pgsns + conditions}
\end{align*}

\begin{align*}
earlier \text{ex.}
\begin{cases}
  x = 0 \}
  x = 1 \{ x = 1 \}
\end{cases}
\begin{cases}
  x = 0 \}
  y = 0 \{ x = y^3 \}
\end{cases}
\end{align*}

\text{If the pgsns are correct, may be the conditions need to be fixed}

\begin{align*}
\begin{cases}
  x = 0 \}
  x = 1 \{ x = 1 \}
\end{cases}
\text{changes \ but not in way that}
\begin{cases}
  x = 0 \}
  x = 1 \{ y = 0 \}
\end{cases}
\end{align*}

\text{Even though the conditions are not disjoint, the annotation is ok}
\{ x=0 \land x_0=0 \} \quad \text{\( x \downarrow = 1 \)} \quad \{ x=1 \}

\{ x_0=0 \} \quad y_0=0 \quad \{ x_0=0, \ y_0=0 \}

\[ x\downarrow = 1 \] not affected by \( y_0=0 \)

\[ x_\downarrow = 0 \] not affected by \( x_\downarrow = 1 \)

\{ x_0=0, y_0=0 \} \quad y \downarrow = y \quad \{ x_\downarrow = 1 \}

\{ x_0=0 \land x_0=0 \} \quad \text{\( x_\downarrow = 1 \)} \quad \{ x_\downarrow = 1 \}

\{ x_0=0 \land x_0=0 \} \quad y_0=0 \quad \{ x_0=0, y_0=0 \}

\[ x\downarrow = 1 \] not changed by \( x_\downarrow = 1 \)

Use parallelism to set
\{ x_0=0, y_0=0 \} \quad \text{\( x_\downarrow = 1 \)} \quad y_\downarrow = y_\downarrow \quad \{ x_\downarrow = 1, y_\downarrow = 0 \}

\[
\text{strengthen}
\]
\{ x_\downarrow = 0, \text{\( x_\downarrow = 1 \)} \quad y_\downarrow = y_\downarrow \quad \{ x_\downarrow = 1, y_\downarrow = 0 \}

\[
\text{weaken}
\]
To check for DPP or Disj. conditions,

\[ \begin{array}{c c c c}
1 & 2 & x & z \\
1 & 3 & y & w \\
2 & 3 & z & w \\
2 & 1 & z & u \\
3 & 1 & w & x \\
3 & 2 & w & z \\
\end{array} \]

So, yes, DPP w/ Disj. Cond.