CS 536 Thu 1/21/21
http://cs.lit.edu/~cse536

More prop logic, proofs, predicate logic

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p \lor q</th>
<th>p \land q</th>
<th>p \rightarrow q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

if p \lor q then p

Logical equivalence

If p (big proposition)
and q

If p \rightarrow q is a tautology
p \Leftrightarrow q they're logically equivalent

If x = y then
you can replace
x by y everywhere
<table>
<thead>
<tr>
<th>$P$</th>
<th>$q$</th>
<th>$p \Rightarrow q$</th>
<th>$q \Rightarrow p$</th>
<th>$p \Leftrightarrow q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
</tbody>
</table>

$2+2=4$  $4 = 3+1$
$5+2=2=3+1$

$\Rightarrow$ is transitive

$\Leftrightarrow$ is transitive

$\iff$ is a tautology
\[ \text{proven: } (p \rightarrow q) \rightarrow q \text{ or neg.of } \]
\[ p \text{ or } (p \rightarrow q) \rightarrow q \rightarrow \top \text{ mutus parsens} \]

**Propositional Logic**

**Predicate Logic**

- Proposition Vars
- Connectives \( \land, \lor, \Rightarrow \)
- Predicates over a Domain: relations \( =, >, < \)
- Axioms for these:
  - \( x + 0 = x \)
  - \( x \times 1 = x \)

**Logic**

- Talk about properties of sets of values
  - \( A \subseteq \)
Universal quantification

Existential quantification

\[ \forall x. x + 0 = x \]
\[ (\forall x) x + 0 = x \]
\[ \forall x (x + 0 = x) \]

\[ \exists x \in \mathbb{Z}. x + y = y \]
\[ \exists x \in \mathbb{R}. x^2 = 1 \]
\[ \exists x \in \mathbb{Q}. x + 0 = x \]

Some predicate

exists some predicate built from plus and equals operators

Witness value

\[ x = 0 \]
\[ x = 1, x = -1 \]

forall x
Syntactic equality = semantic equality

$x = x$  \[\text{Semantic equality} \iff \text{equality of values/meanings}\]

Syntactic $x + 0 = x + 0$  \[\text{text}\]

Parenthesization is verifiable w/ simple algorithm

Syntactic equality implies semantic equality

$2 + 1 \neq 3$  \[\text{ Due to implicit values } e_1 = e_2 \]

$2 + 1 = 3$  \[\text{ Due to associative, commutative} \]

Converse doesn't hold always

maybe verifiable using expensive alg.

no algorithm exists
For us, we'll use \( = \) meaning syntactically equal.

Redundant ()

\[
(x*y)+z = x*(y+z)
\]

\(x*(y+z) \neq (x*y)+z\)

\(x*(y+z) \neq x*y+z\)

\((x+y)+z \neq x+(y+z)\)

\(e_1 + e_2\) have vars and opts in different order; then \(e_1 \neq e_2\)

\(e_1 = e_2\) must have vars and opts in the same order.

\(2+1 \neq 1+2\)

Not commutativity.
Quantifiers?

Quantifier is necessary.

\( \forall x. (\exists y. \text{predicate}) \)

\( \exists y. (\forall x. \text{predicate}) \)

\( \exists y. (\forall x. \text{predicate}) \Rightarrow (\forall x. \text{predicate}) \Rightarrow (\exists y. \text{predicate}) \)

\( \forall x. (\exists y. \text{predicate}) \Rightarrow (\forall x. \text{predicate}) \Rightarrow (\exists y. \text{predicate}) \)