Strength, Weakest Preconditions, Syntactic Substitution

CS 536: Science of Programming, Fall 2018
Due Wed Oct 17, 11:59 pm
Late HW due by noon Oct 18 because Exam 2 is on Oct 22

10/16: p.1

A. Why?

B. Objectives

At the end of this homework, you should be able to

C. Problems [100 points total]

Part 1: Strength (30 points)

1. [6 = 2*3 points] Let \( p_0 \rightarrow p_1, p_1 \rightarrow p_2, q_0 \rightarrow q_1, \) and \( q_1 \rightarrow q_2 \) all be valid. From \( \{ p_1 \} S \{ q_1 \} \), there are four triples we get by replacing \( p_1 \) by \( p_0 \) or \( p_2 \) and \( q_1 \) by \( q_0 \) or \( q_2 \).
   a. If \( \sigma \models \{ p_1 \} S \{ q_1 \} \), which of the four triples are also satisfied by \( \sigma \) (under \( \models \))? Briefly justify.
   b. If \( \sigma \models_{tot} \{ p_1 \} S \{ q_1 \} \), which of the four triples are also satisfied by \( \sigma \) (under \( \models_{tot} \))? Briefly justify.

2. [24 = 8*3 points] Let \( S \) be deterministic, let \( w \Leftrightarrow wp(S, q) \) and let \( p \) be arbitrary (not necessarily \( \Rightarrow, \Leftarrow, \) or \( \Leftrightarrow \)). For each of the following triples, assume \( \sigma \models \) the precondition. Does this imply \( \sigma \models \) or \( \not\models \) the triple? How about \( \sigma \models_{tot} \) or \( \not\models_{tot} \) the triple? Just say which; you don’t have to write out your reasoning\(^1\) (but you should be able to if asked in the future. (And don’t forget, \( \sigma \models_{tot} \ldots \) implies \( \sigma \models \ldots \))
   a. \( \{ p \land w \} S \{ q \} \)
   b. \( \{ p \land \neg w \} S \{ q \} \)
   c. \( \{ \neg p \land w \} S \{ q \} \)
   d. \( \{ \neg p \land \neg w \} S \{ q \} \)
   e. \( \{ p \land w \} S \{ \neg q \} \)
   f. \( \{ p \land \neg w \} S \{ \neg q \} \)
   g. \( \{ \neg p \land w \} S \{ \neg q \} \)
   h. \( \{ \neg p \land \neg w \} S \{ \neg q \} \)[10/16]

Part 2: Weakest preconditions (40 points)

For the following problems, unless otherwise asked, do syntactic calculations but no arithmetic or logical simplifications.

3. [10 = 2*5 points] Calculate the \( wp \) in each of the following cases. (Just calculate the syntactically \( wp \); don’t also logically simplify the result.)
   a. \( wp(n := n \ast (n-k), \ n < k \ast n) \)
   b. \( wp(x := x \ast y \land y := y-x, \ x^2 > y^2) \)

4. [30 = 5*6 points] For each of the following, calculate the \( wp \) of the statement and postcondition and then simplify it logically to something you find pleasing to use for \( p \). (Note: calculate first, simplify second.)
   a. \( \{ p \} \ j := j+i; \ i := i-k \} \ i \leq j \land j-i < n+k \)

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\(^1\) You should be able to write out the reasoning; we’re just not requiring it. Also, you should know the answers if \( S \) is nondeterministic.
b. \( \{ p \} j := i \times j; k := j \times i + k \{ 0 < i < j < k \} \)

c. \( \{ p \} \text{if } x \geq 0 \text{ then } x := x + k \text{ else } y := y - k \text{ fi } \{ x > y \} \)

d. \( \{ p \} \text{if } b[M] \leq v \text{ then } L := M \text{ else } R := M \text{ fi } \{ L < R \land b[L] \leq v < b[R] \} \)

e. \( \{ p \} \text{if } x < 0 \text{ then } x := 2 - x \text{ else if } x < 2 \text{ then } x := x + 2 \text{ fi } \{ x^2 > x \} \)

(Don’t forget the implicit "\textbf{else skip}" clause.)

**Part 3: Syntactic Substitution [30 points]**

\[ [30 = 5 \times 6 \text{ points}] \text{ Let } p \equiv x \times y < f(a) \lor \forall x . x \geq a \times y \rightarrow \exists y . x \div y > y - a - z. \text{ Calculate the following substitutions. (Do syntactic calculations but no arithmetic or logical simplifications.)} \]

\[ a. \quad p[y - z/x]. \quad b. \quad p[y + z/y] \quad c. \quad p[x \times y/a] \]

\[ d. \quad p[y \div a/a] \quad e. \quad p[x + y/a][y - z/x] \]