A. Why?
- Runtime errors cause failure of normal program execution.
- Nondeterminism can help us develop programs without worrying about overlapping cases.
- Correctness triples are how we write a program with its specification.

B. Objectives
At the end of this homework, you should be able to
- Say when and how evaluation of an expression or program fails due to a runtime error.
- Evaluate a nondeterministic if-fi and do-od.
- Identify the properties that connect satisfaction of partial and total correctness triples with satisfaction of preconditions and postconditions and with the denotational semantics of programs.

C. Problems [100 points total]

Part 1: Runtime Errors (30 points)
For Questions 1 – 4, calculate the denotational semantics \( M(\text{statement}, \sigma) \) using the denotational semantics rules. Recall that integer division and \( \sqrt{\ldots} \) truncate. If the meaning is some flavor of \( \perp \), be specific and say \( \perp_d \) or \( \perp_e \).

1. [5 points] Calculate \( M(\text{if } x \text{ odd then } x := x-1 \text{; } x := x/2, \sigma) \) where \( \sigma \) is an arbitrary state. Your answer will be symbolic and have two cases. Define \( x \text{ odd } \equiv x \mod 2 = 1 \).

2. [5 points] Calculate \( M(W, \{ x = 0, y = 1 \}) \) where \( W \equiv \text{while } x \neq 3 \text{ do } S \text{ od} \) and \( S \equiv x := x+1; y := y+y \). Use the technique of characterizing the sequence of states seen at the loop test. It may help if you calculate \( M(S, \tau) \) for an arbitrary \( \tau \).

3. [5 points] Calculate \( M(W, \{ x = 4, y = 1 \}) \) where \( W \) is as in the previous problem.

4. [15 = 3 * 5 points] Calculate \( M(S, \sigma) \) where \( S \equiv v := b[x]; w := x/v; y := \sqrt{w}, \sigma = \{ b = (3, 0, -2), x = \alpha \} \), and a value for \( \alpha \) as below.
   a. \( \alpha = -1 \)
   b. \( \alpha = 0 \)
   c. \( \alpha = 1 \)

Part 2: Nondeterminism (20 points)

5. [5 points] What are the semantic similarities and differences among \( IF_1, IF_2, \) and \( IF_3 \) where
   - \( IF_1 \equiv \text{if } B_1 \rightarrow S_1 \text{; } B_2 \rightarrow S_2 \text{ fi} \)
   - \( IF_2 \equiv \text{if } B_2 \rightarrow S_2 \text{; } B_1 \rightarrow S_1 \text{ fi} \) and
   - \( IF_3 \equiv \text{if } B_1 \rightarrow S_1 \text{; } B_2 \rightarrow S_2 \text{; } \neg B_1 \land \neg B_2 \rightarrow \text{skip fi} \)
6. [5 points] Let \( T \_ \text{or} \_ F() \) be a function that nondeterministically returns true or false, and let \( \text{error}() \) be a function that causes a runtime error. Implement \( \text{if } B_1 \rightarrow S_1 \sqcup B_2 \rightarrow S_2 \text{fi} \) using these functions along with regular deterministic \( \text{if-else} \).

7. [10 = 3 + 7 points] Let \( IF \equiv \text{if } x \neq 0 \rightarrow x := x+1; y := y+1 \sqcup x \neq 0 \rightarrow x := x-1; y := y+1 \text{fi} \). Let \( DO \) be the corresponding loop \( DO \equiv \text{while } x \neq 0 \text{ do } IF \text{ od} \). Let \( \sigma_0 \) be the state \( \{ x = 1, y = 0 \} \).
   a. For an arbitrary state \( \tau \), what states make up \( M(IF, \tau) \)? Show your calculations.
   b. What states make up \( M(DO, \sigma_0) \)? (You can justify your answer informally — the calculations are messy).

**Part 3: Hoare Triples [50 points]**

For all these questions, write a short answer, at most a paragraph. As the default, assume \( \sigma \in \Sigma \) and \( S \) might cause an error (\( \bot \in M(S, \sigma) \) is possible).

8. [3 points] Why are \( \models \{ p \} S \{ q \} \) and \( \models_{\text{tot}} \{ p \} S \{ q \} \) equivalent if \( S \) cannot cause an error?

9. [3 points] If \( \sigma \models_{\text{tot}} \{ p \} S \{ \top \} \) and \( M(S, \sigma) \subsetneq \Sigma \), can we conclude anything about \( \sigma \)?

10. [4 = 2 * 2 points] Can \( \sigma \not\models_{\text{tot}} \{ p \} S \{ q \} \) and \( \sigma \not\models_{\text{tot}} \{ p \} S \{ \neg q \} \) occur simultaneously?
   a. When \( S \) is deterministic?
   b. When \( S \) is nondeterministic but always halts?

Problems 11 - 17 are all written briefly as "Does \( X \) imply \( Y \)? Not \( Y \)? Neither?" Explain briefly why \( Y \) must hold or why not \( Y \) must hold or why both \( Y \) and not \( Y \) are possible.

11. [3 points] Does \( \sigma \models \{ p \} S \{ q \} \) imply \( \sigma \models { p } \? \sigma \not\models \neg p \? \? \) Neither?T

12. [3 points] Does \( \sigma \not\models \{ p \} S \{ q \} \) imply \( \sigma \models \{ p \} S \{ \neg q \} \)? \( \sigma \not\models \{ p \} S \{ \neg q \} \)? Neither?

13. [8 = 4 * 2 points] Does \( \sigma \not\models \{ p \} S \{ q \} \) imply
   a. \( \sigma \models p \? \sigma \not\models \neg p \? \) Neither?
   b. \( M(S, \sigma) \subseteq \Sigma \? M(S, \sigma) \not\subseteq \Sigma \? \) Neither?
   c. \( M(S, \sigma) \models q \? M(S, \sigma) \not\models q \? \) Neither?
   d. \( M(S, \sigma) \models \neg q \? M(S, \sigma) \not\models \neg q \? \) Neither?

14. [6 = 3 * 2 points] Do \( \sigma \models p \) and \( \sigma \models \{ p \} S \{ q \} \) together imply
   a. \( M(S, \sigma) \subseteq \Sigma \? M(S, \sigma) \not\subseteq \Sigma \? \) Neither?
   b. \( M(S, \sigma) \models q \? M(S, \sigma) \not\models q \? \) Neither?
   c. \( M(S, \sigma) \models \neg q \? M(S, \sigma) \not\models \neg q \? \) Neither?

15. [6 = 3 * 2 points] Do \( \sigma \models p \) and \( \sigma \models_{\text{tot}} \{ p \} S \{ q \} \) together imply
   a. \( M(S, \sigma) \subseteq \Sigma \? M(S, \sigma) \not\subseteq \Sigma \? \) Neither?
   b. \( M(S, \sigma) \models q \? M(S, \sigma) \not\models q \? \) Neither?
   c. \( M(S, \sigma) \models \neg q \? M(S, \sigma) \not\models \neg q \? \) Neither?

16. [8 = 4 * 2 points] Does \( \sigma \not\models_{\text{tot}} \{ p \} S \{ q \} \) imply \( \neg \) [10/5] Added missing paren to part b.
   a. \( M(S, \sigma) \subseteq \Sigma \? M(S, \sigma) \not\subseteq \Sigma \? \) Neither?
   b. \( M(S, \sigma) \models q \? M(S, \sigma) \not\models q \? \) Neither?
   c. \( M(S, \sigma) \models \neg q \? M(S, \sigma) \not\models \neg q \? \) Neither?

17. [6 = 3 * 2 points] Do \( \sigma \models \{ p \} S \{ q \} \) and \( \sigma \not\models_{\text{tot}} \{ p \} S \{ q \} \) together imply
   a. \( M(S, \sigma) \subseteq \Sigma \? M(S, \sigma) \not\subseteq \Sigma \? \) Neither?
   b. \( M(S, \sigma) \models q \? M(S, \sigma) \not\models q \? \) Neither?
   c. \( M(S, \sigma) \models \neg q \? M(S, \sigma) \not\models \neg q \? \) Neither?