Weakest Preconditions and Substitution

Class 10: Weakest Preconditions, part 1 [18 points]

1. [3 points] For nondeterministic if, say IF \( N \equiv \text{if } B_1 \rightarrow S_1 \square B_2 \rightarrow S_2 \text{ fi} \), then \( \text{wp}(IF, q) = (B_1 \rightarrow \text{wp}(S_1, q)) \land (B_2 \rightarrow \text{wp}(S_2, q)) \). Is it also the case \( \text{wp}(IF, q) \equiv (B_1 \land \text{wp}(S_1, q)) \lor (B_2 \land \text{wp}(S_2, q)) \)? Explain briefly.

For Problems 2 - 4, calculate the \( \text{wp} \) in each of the following cases. (Just do the syntactic calculations; don't simply the result. E.g., \( \text{wp}(x := 2, x \cdot x = 4) = 2 \cdot 2 = 4 \), not \( T \).)

2. [5 points] \( \text{wp}(m := m + f(m,y), f(m,y) < g(y,m)) \)

3. [5 points] \( \text{wp}(u := u \cdot k; k := u, u > h(k)) \)

4. [5 points] \( \text{wp}(\text{if } x < 0 \text{ then } x := -x, x^2 \geq x) \). (Don't forget the implicit "else skip" clause.)

Class 11: Weakest Preconditions part 2 and Domain Predicates [18 points]

Calculate the \( \text{wp} \)'s below. Show some detail if you want partial credit for a wrong answer. For logical simplifications, drop redundant conjuncts from predicates. For reference, 
\[
\text{wp}(S, q) = D(wlp(S, q)) \land wlp(S, q) \land D(S)
\]
\[
\text{wp}(\text{if } B \text{ then } S_1 \text{ else } S_2 \text{ fi}, q) = D(B) \land (B \rightarrow \text{wp}(S_1, q)) \land (\neg B \rightarrow \text{wp}(S_2, q))
\]
\[
D(S_1; S_2) = D(S_1) \land \text{wp}(S_1, D(S_2))
\]

5. [7 points] \( \text{wp}(\text{if } z \geq 0 \text{ then } x := x + a/y \text{ else } x := y + b/x \text{ fi}, a \leq x < f(x,y)) \)

6. [6 points] \( \text{wp}(x := b[\text{sqrt}(y)], x > 0) \)

7. [5 points] \( \text{wp}(k := k - b[k], k \neq 0) \)

Class 12: Syntactic Substitutions [16 points]

For Problems 8 - 10, let \( p = x + y < f(a) \lor \exists x . x \geq a + y \rightarrow \exists y . x \cdot y > b - y - c \) and calculate the substitutions below. Show some detail if you want partial credit for a wrong answer. (Just do the syntactic calculations; not any arithmetic or logical simplifications.)

8. [3 points] \( p[y \cdot z/y] \)

9. [5 points] \( p[a\cdot y/a] \)

10. [8 points] \( p[x\cdot y/a][y\cdot z/x] \)