Hoare Triples pt.2

Class 9: Hoare Triples, pt 2

1. Backward assignment tells us \{P(e)\} x := e {P(x)} where e = b*b - 4*a*c and \(P(x) = 0 \leq x \to \sqrt{x}\) is defined. \(P(e) = 0 \leq e \to \sqrt{e}\) is defined.

2. Certainly \(x := n; y := m\) establishes \(x = n \land y = m\), so \(x*y = n*m\). To get \(1 \leq x*y \leq n*m\), since we can’t infer \(1 \leq n*m\) with what we know, we need to assume it. We can use \(p = 1 \leq n*m\).

3. (Weaken/strengthen pre-/post-conditions)
   1a. \(\sigma \models \{p_0\} S \{q_2\}\) -- we can always strengthen preconditions and weaken postconditions.
   1b. \(\sigma \models_{tot} \{p_0\} S \{q_2\}\) for the same reason.

4. We’re given \(\sigma \models \{p_1\} S \{q_1\}\) and \(\sigma \models \{p_2\} S \{q_2\}\).
   a. Yes, \(\sigma \models \{p_1 \land p_2\} S \{q_1 \land q_2\}\). Since \(\sigma\) satisfies \(p_1 \land p_2\), it satisfies just \(p_1\) and \(p_2\) separately, so our two triples tells us that \(M(S, \sigma) - \bot \models q_1\) and \(q_2\) separately, so it also satisfies \(q_1 \land q_2\).
      So \(\sigma \models \{p_1 \land p_2\} S \{q_1 \land q_2\}\).
   b. We can’t infer \(\sigma \models \{p_1 \lor p_2\} S \{q_1 \land q_2\}\). If \(\sigma \models p_1 \lor p_2\), then it’s possible to have \(\sigma \models p_1\), but \(\sigma \not\models p_2\) or both. From \(\sigma \models p_1\), we can infer \(M(S, \sigma) - \bot \models q_1\), but \(\sigma \not\models p_2\) doesn’t tell us \(M(S, \sigma) - \bot \models q_2\). Without it, we don’t know \(M(S, \sigma) - \bot \models q_1 \land q_2\), so we don’t know \(\sigma \models \{p_1 \lor p_2\} S \{q_1 \land q_2\}\).
   c. Yes, \(\sigma \models \{p_1 \lor p_2\} S \{q_1 \lor q_2\}\). If \(\sigma \models p_1\), then we know \(\sigma \models \{p_1\} S \{q_1\}\), which implies \(\sigma \models \{p_1\} S \{q_1 \lor q_2\}\). Similarly, if \(\sigma \models p_2\), we also get to know \(\sigma \models \{p_2\} S \{q_1 \lor q_2\}\). Since \(\sigma \models p_1 \lor p_2\) requires \(\sigma\) to satisfy \(p_1\) or \(p_2\), we can conclude \(\{p_1 \lor p_2\} S \{q_1 \lor q_2\}\).
   d. Yes. From part (a) above, we know \(\sigma \models \{p_1 \land p_2\} S \{q_1 \land q_2\}\), and since \(q_1 \land q_2\) implies \(q_1 \lor q_2\), we know \(\sigma \models \{p_1 \land p_2\} S \{q_1 \lor q_2\}\).
Hoare Triples pt.2

CS 536: Science of Programming, Spring 2021
Due Sun Feb 28, 11:59 pm  Post solution Mon Mar 1

Preliminary Exam 1

- Exam 1 is Thu Mar 4 and covers classes 1 – 9 (i.e., up through Hoare Triples pt.2). Mostly multiple choice but some written problems too.

Notes
- This homework is only 25 points (it covers just one class, not two).
- This homework is due on Sun Feb 28; no late submissions allowed so that I can can post a solution Monday Mar 1. (There isn't time to grade and return this homework before the exam.)

Problems [25 points total]

Class 9: Hoare Triples, pt 2 (25 points)

1. [3 points] Study the triple \{???\} \(x := b*b - 4*a*c\ \{0 \leq x \rightarrow \sqrt{x}\) is defined\}. Using backward assignment, what can we use for the precondition of the triple?

2. [4 points] Study the two triples \(\{p\} x := n; y := m\ \{p \land x = n \land y = m\}\) and \(\{1 \leq x*y \leq n*m\\} S \{q\}\). Find a predicate \(p\) that makes it possible to join the two triples into a sequence.

3. [6 = 2*3 points] Let \(p_0 \rightarrow p, p \rightarrow p_1, q_0 \rightarrow q, \) and \(q \rightarrow q_1\) all be valid. From \(\{p\} S \{q\}\), there are four triples of the form \(\{p_i\} S \{q_j\}\) that get by replacing \(p\) by \(p_0\) or \(p_1\) and \(q\) by \(q_0\) or \(q_1\).
   a. If \(\sigma \models \{p\} S \{q\}\), which of the four triples \(\sigma \models \{p_i\} S \{q_j\}\) is/are also satisfied by \(\sigma\) (under \(\models\))? Briefly justify.
   b. If \(\sigma \models_{\text{tot}} \{p\} S \{q\}\), which of the four triples \(\sigma \models \{p_i\} S \{q_j\}\) is/are also satisfied by \(\sigma\) (under \(\models_{\text{tot}}\))? Briefly justify.

4. [12 = 4*3 points] Say \(\sigma \models \{p_1\} S \{q_1\}\) and \(\sigma \models \{p_2\} S \{q_2\}\).
   a. Does \(\sigma \models \{p_1 \land p_2\} S \{q_1 \land q_2\}\)? Justify briefly.
   b. Does \(\sigma \models \{p_1 \lor p_2\} S \{q_1 \land q_2\}\)? Justify briefly.
   c. Does \(\sigma \models \{p_1 \lor p_2\} S \{q_1 \lor q_2\}\)? Justify briefly.
   d. Does \(\sigma \models \{p_1 \land p_2\} S \{q_1 \lor q_2\}\)? Justify briefly.