Weakest Preconditions and Substitution

CS 536: Science of Programming, Fall 2020

Due Sat Nov 14, 11:59 pm

Class 10: Weakest Preconditions, part 1 [18 points]

1. [3 points] For nondeterministic if, say IF N ≡ if B₁ → S₁ ∨ B₂ → S₂ ∨ i, then wp(IF N, q) = (B₁ → wp(S₁, q)) ∧ (B₂ → wp(S₂, q)). Is it also the case (B₁ → wp(S₁, q)) ∨ (B₂ → wp(S₂, q))?

Explain briefly.

For Problems 2 - 4, calculate the wp in each of the following cases. (Just do the syntactic calculations; don’t simply the result. E.g., wp(x := 2, x*x := 4) = 2*2 = 4, not T.)

2. [5 points] wp(m := m + f(m, y), f(m, y) < g(y, m))

3. [5 points] wp(u := u * k; k := u, u > h(k))

4. [5 points] wp(if x < 0 then x := -x, else S₂) (Don’t forget the implicit "else skip" clause.)

Class 11: Weakest Preconditions part 2 and Domain Predicates [18 points]

Calculate the wp's below. Show some detail if you want partial credit for a wrong answer. For logical simplifications, drop redundant conjuncts from predicates. For reference,

wp(S, q) = D(wp(S, q)) ∧ wp(S, q) ∧ D(S)
wp(if B then S₁ else S₂, q) = D(B) ∧ (B → wp(S₁, q)) ∧ (¬B → wp(S₂, q))
D(S₁ ; S₂) = D(S₁) ∧ wp(S₁, D(S₂))

5. [7 points] wp(if z ≥ 0 then x := x + a/y else x := y + b/x, a ≤ x < f(x, y))

6. [6 points] wp(x := b[sqrt(y)], x > 0)

7. [5 points] wp(k := k - b[k], k ≠ 0)

Class 12: Syntactic Substitutions [16 points]

For Problems 8 - 10, let p = x + y < f(a) ∨ ∃x . x ≥ a + y → ∃y . x*y > b-y-c and calculate the substitutions below. Show some detail if you want partial credit for a wrong answer. (Just do the syntactic calculations; not any arithmetic or logical simplifications.)

8. [3 points] p[y*z/y]

9. [5 points] p[a-y/a]

10. [8 points] p[x*y/a][y-z/x]