Hoare Triples, wp/wlp, Syntactic Substitution
CS 536: Science of Programming, Fall 2019
Due Wed Oct 16

10/6

Problems [50 points]

Lectures 8 & 9: Hoare Triples [18 points]

1. [4 points] Let \( \sigma \models \{ p \} S \{ q \} \). If \( \sigma \not\models p \), do we know whether \( M(S, \sigma) \) contains \( \bot \) or not? If \( M(S, \sigma) - \{ \bot \} \models q \) or not? Repeat, if \( \sigma \models p \).

2. [2 points] Briefly, how are \( \models \{ p \} S \{ q \} \), \( \models_{\text{tot}} \{ p \} S \{ q \} \), and \( \models_{\text{tot}} \{ p \} S \{ \tau \} \) related?

3. [2 points] If \( \sigma \models_{\text{tot}} \{ p \} S \{ \tau \} \) and \( \bot \in M(S, \sigma) \), what is the relationship between \( \sigma \) and \( p \)?

4. [4 points] If \( \sigma \not\models \{ p \} S \{ q \} \) and \( S \) is deterministic, do we know whether \( \sigma \models p \) or not? \( M(S, \sigma) \models q \) or \( \neg q \)? \( \bot \in \sigma \) or \( \not\in M(S, \sigma) \)? What if \( S \) is nondeterministic?

5. [3 points] What are the relationships between \( \sigma \models (or \not\models) \{ p \} S \{ q \} \), \( \sigma \models (or \not\models) \{ p \} S \{ \neg q \} \), \( \sigma \models_{\text{tot}} (or \not\models_{\text{tot}}) \{ p \} S \{ q \} \), and \( \sigma \models_{\text{tot}} (or \not\models_{\text{tot}}) \{ p \} S \{ \neg q \} \)?

6. [3 points] Suppose \( S \) is deterministic and \( \sigma \not\models_{\text{tot}} \{ p \} S \{ q \} \). Can we conclude anything about \( \sigma \models (or \not\models) \{ p \} S \{ q \) or \( \neg q \)\)? (Break down your analysis into cases with \( \bot \in \sigma \) or \( \not\in M(S, \sigma) \).)

Lectures 10 & 11: wp and wlp [20 points]

7. [3 points] For nondeterministic if, say \( \text{IF}_N \equiv \text{if } B_1 \rightarrow S_1 \sqcup B_2 \rightarrow S_2 \text{ fi} \), the basic calculation is \( \text{wp(\text{IF}_N, q)} \equiv (B_1 \rightarrow \text{wp}(S_1, q)) \land (B_2 \rightarrow \text{wp}(S_2, q)) \). Is it also the case \( \text{wp}(\text{IF}_N, q) \equiv (B_1 \land \text{wp}(S_1, q)) \lor (B_2 \land \text{wp}(S_2, q)) \)? Explain briefly.

8. [3 points] Can we always strengthen preconditions or weaken postconditions? Give an example of when it’s useful and when it’s not useful.

9. [4 points] Which of the following (four) statements behave differently depending on whether \( S \) is deterministic or nondeterministic. Explain briefly.
   - \( \text{wp}(S, p \lor q) \rightarrow \text{wp}(S, p) \lor \text{wp}(S, q) \)
   - \( \text{wp}(S, p \land q) \rightarrow \text{wp}(S, p) \land \text{wp}(S, q) \)

10. [4 points] Let \( p_0 \rightarrow w \rightarrow p_1 \) where \( w \equiv \text{wp}(S, q) \). Which of the following properties can fail? Explain briefly.
   - \( \{ p_0 \} S \{ q \}, \{ p_1 \} S \{ q \}, \{ \neg p_0 \} S \{ \neg q \}, \{ \neg p_1 \} S \{ \neg q \} \)

11. [6 points] Calculate the \( \text{wlp} \) or \( \text{wp} \) requested each of the following cases. Just do the syntactic calculation; don’t also logically simplify the result.
   a. \( \text{wlp}(x := x + y; y := x*z+y, x - y - z < f(x, y, z)) \)
   b. \( \text{wlp}\{\text{if } x \geq y \text{ then } x := x-y \text{ fi}; y := f(f(x/2, y), x-y), x < y \} \)
   c. \( \text{wp}\{\text{if } x \geq y \text{ then } x := x-y \text{ fi}; y := f(f(x/2, y), x*y), x < y \} \). Assume \( D(f(u, v)) \equiv u > v \)
Lecture 12: Syntactic Substitution [12 points]

12. [12 points] Let $p \equiv (z < 2 \times x \lor x \leq y) \land (\exists x. x \div y > y \div z) \land (\exists y. g(z^2 + z) < x \times y)$. For the calculations below, show some detail if you want partial credit for a wrong answer. Do not logically simplify the results.

   b. [3 points] Calculate $p[(z+a)/z]$.
   c. [6 points] Calculate $p[x+y/z]$. 

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HW 4: Lect 8 – 12