Sequential Nondeterminism; Hoare Triples

CS 536: Science of Programming, Fall 2020

Due Sat Oct 31, 11:59 pm

A. Class 7: Sequential Nondeterminism [14 points]

1. [5 points] Translate the nondeterministic
   \[ if \ B_1 \to S_1 \Box \ B_2 \to S_2 \fi \]
   to an equivalent deterministic \( if - else \) augmented with a function \( \text{maybe}(\cdot) \) that nondeterministically returns true or false and a function \( \text{error}(\cdot) \) that causes a runtime error.

2. [4 points] Translate the nondeterministic loop
   \[ do \ B_1 \to S_1 \Box \ B_2 \to S_2 \od \]
   to an equivalent loop that uses \( \text{while} \ B_1 \lor B_2 \ do ... \ od \)

3. [5 points] Let \( DO \equiv \ \text{do} \ x \neq 0 \to x := x-1; y := y+1 \Box \ x \neq 0 \to x := x-1; y := y-1 \od \). If we run \( DO \) starting in state \( \{ x = \beta, y = \delta \} \) (where \( 0 \leq \beta \)), what is the set of values \( y \) could have on completion?

B. Class 8: Hoare Triples, pt 1 [20 points]

The questions below have the form “If \( X \), then \( Y \) _____ occur”. To answer them, fill in the blank with “must”, “can’t”, or “may or may not”.

- **Must occur** means \( X \) implies \( Y \). (E.g., if \( x > 1 \), then \( x > 0 \) must occur.)
- **Can’t occur** means \( X \) implies \( \neg Y \). (E.g., if \( x > 1 \), then \( x < -3 \) can’t occur.)
- **May or may not occur** means \( Y \) does not depend on \( X \): Either \( X \land Y \) or \( X \land \neg Y \) can hold. (E.g., if \( x > 1 \), then \( y = 0 \) may or may not occur.)

You’re not required to justify your answer, though you can if you want to (and you should be able to if asked to in an exam).

[20 = 10 * 2 points] Assume throughout that \( \sigma \neq \bot \) and that \( S \) might be deterministic or nondeterministic unless said otherwise

1. If \( \sigma \models \{ p \} S \{ q \} \) and \( \sigma \neq p \), then \( \bot \in M(S, \sigma) \) _____ occur.
2. If \( \sigma \models \{ p \} S \{ q \} \) and \( \sigma \models p \), then \( \bot \in M(S, \sigma) \) _____ occur.
3. If \( \sigma \models \{ p \} S \{ q \} \) and \( \sigma \models p \), then \( M(S, \sigma) \setminus \{ \bot \} \models q \) _____ occur.
4. If \( \equiv_{\text{tot}} \{ p \} S \{ q \} \) then \( \equiv_{\text{tot}} \{ p \} S \{ T \} \) _____ occur.
5. If \( \bot \notin M(S, \sigma) \), \( M(S, \sigma) \neq q \), and \( S \) is deterministic, then \( M(S, \sigma) \models \neg q \) _____ occur.
6. If \( \bot \notin M(S, \sigma) \), \( M(S, \sigma) \neq q \), and \( S \) is nondeterministic, then \( M(S, \sigma) \models \neg q \) _____ occur.
7. If \( \sigma \models \{ p \} S \{ q \} \), then \( \sigma \models \{ p \} S \{ \neg q \} \) _____ occur.
8. If \( \sigma \neq_{\text{tot}} \{ p \} S \{ q \} \) and \( S \) is deterministic, then \( \sigma \models \{ p \} S \{ \neg q \} \) _____ occur.
9. If $\sigma \not\models \{p\} S \{q\}$ and $S$ is deterministic, then $\sigma \models_{tot} \{p\} S \{\neg q\}$ _____ occur.
10. If $\sigma \not\models \{p\} S \{q\}$ and $S$ is non-deterministic, then $\sigma \models_{tot} \{p\} S \{\neg q\}$ _____ occur.

C. Class 9: Hoare Triples, pt 2 [16 points]

1. [8 = 2*4 points] Let $p_0 \to p$, $p \to p_1$, $q_0 \to q$, and $q \to q_1$ all be valid. From $\{p\} S \{q\}$, there are four triples we get by replacing $p$ by $p_0$ or $p_1$, and $q$ by $q_0$ or $q_1$.
   a. If $\sigma \models \{p\} S \{q\}$, which of the four triples is/are also satisfied by $\sigma$ (under $\models$)? Briefly justify.
   b. If $\sigma \models_{tot} \{p\} S \{q\}$, which of the four triples is/are also satisfied by $\sigma$ (under $\models_{tot}$)? Briefly justify.

2. [8 = 4*2 points] Say $\sigma \models \{p_1\} S \{q_1\}$ and $\sigma \models \{p_2\} S \{q_2\}$.
   a. Does $\sigma \models \{p_1 \land p_2\} S \{q_1 \land q_2\}$? Justify briefly.
   b. Does $\sigma \models \{p_1 \lor p_2\} S \{q_1 \land q_2\}$? Justify briefly.
   c. Does $\sigma \models \{p_1 \lor p_2\} S \{q_1 \lor q_2\}$? Justify briefly.
   d. Does $\sigma \models \{p_1 \land p_2\} S \{q_1 \lor q_2\}$? Justify briefly.