Language Syntax, Semantics, Errors, Nondeterminism

CS 536: Science of Programming, Fall 2019
Due Wed 9/25, 11:59 pm

Problems [50 points]

Lecture 5: Language Syntax/Operational Semantics

1. [8 points] Translate the program below into our programming language.

   \[
   \begin{align*}
   x &= 1; \ j = 0; \ while \ (j++ <= m) \ x++; \ x += y;
   \end{align*}
   \]

2. [8 = 2*4 points] Let \( S \equiv if \ x > 0 \ then \ x := x + 2 \cdot y; \ y := 3 \cdot y fi. \)
   a. Evaluate \( \langle S, \{ x = 2, \ y = 6 \} \rangle \) to completion, using step-by-step operational (i.e., \( \rightarrow \)) semantics.
   b. Evaluate \( \langle S, \{ x = -2, \ y = 8 \} \rangle \) to completion, using step-by-step operational semantics.

3. [10 points] Let \( W \) be the program below and let \( \sigma_0 = \{ i = 1, \ x = 1, \ n = 5 \}. \)

   \[
   W = \text{while } i < n \ do \ S \ od \text{ where } S = i := i + 1; \ x := x \cdot i \cdot i
   \]

   Evaluate \( \langle W, \sigma_0 \rangle \) to completion. You can use \( \rightarrow^n \) to emphasize how each iteration changes the state, but be sure to include six configurations (which ones are your choice), including the initial \( \langle W, \sigma_0 \rangle \) and final \( \langle E, (you \ fill \ in) \rangle \).

Lecture 6: Denotational Semantics, Runtime Errs, Sequential Nondeterminism pt. 1

4. [6 = 2*3 points] What are the denotational semantics of the configurations in Problems 2a and 2b? (i.e., \( M(S, \ldots) = \ldots \)?)

5. [3 points] Let \( W \) be the loop in Problem 3. What is the set of \( \sigma \) such that \( \langle W, \sigma \rangle \rightarrow^* \langle E, \bot \rangle? \)

6. [6 = 2*3 points] Let \( S \) be deterministic.
   a. If \( \bot \in M(S, \sigma) \), can we conclude anything about \( \langle S;\ T, \sigma \rangle \) where \( T \) is any other statement? Give a brief justification.
   b. If \( \langle S, \sigma \rangle \rightarrow^* \langle E, \tau \rangle \) and \( \tau \neq T \), can we conclude anything more specific about \( \tau \)? Give a brief justification. (Recall \( T \) means true.)
Lecture 7: Sequential Nondeterminism pt. 2

7. [3 points] Let $S$ be nondeterministic. Suppose $S$ always terminates when run in $\sigma$ (i.e., $\bot \notin M(S, \sigma)$). Is it possible nonetheless for there to be a predicate $\varphi$ where $M(S, \sigma) \not\equiv \varphi$ and $M(S, \sigma) \not\equiv \neg \varphi$ simultaneously? Give a brief explanation.

8. [6 = 2*3 points] Let $W$ be the incomplete program $\text{do } x \geq 0 \rightarrow \ldots \text{□ } x \ldots \rightarrow \ldots \text{od}$. Complete $W$ by completing the guarded commands such that both a and b below are possible (and say what $W$ is).
   a. $\bot, \in M(W, \{x = 0\})$. Sketch an operational execution path where this happens. (I.e., $\langle W, \{x = 0\} \rangle \rightarrow^* \text{some configuration} \rightarrow^* \text{some configuration}$ and so on.
   b. $\{x = \alpha\} \in M(W, \{x = 0\})$. Sketch an operational execution path where this happens and say what value $\alpha$ has.