



Well-typed programs don't get stuck

Typing judgment  $\Gamma \vdash E : \tau$

Context maps vars to types

e.g.  $\emptyset \quad x:\text{unit}, y:\text{unit} \rightarrow \text{unit}$

we write  $\Gamma, x:\tau$  for  $\Gamma$  extended w/  $x:\tau$

UNIT  $\frac{}{\Gamma \vdash () : \text{unit}}$   
 $\uparrow$   
 any context

VAR  $\frac{}{\Gamma, x:\tau \vdash x:\tau}$   
 any ctx w/  $x:\tau$

ABS  $\frac{\Gamma, x:\tau \vdash E : \tau'}{\Gamma \vdash \lambda x:\tau. E : \tau \rightarrow \tau'}$

APP  $\frac{\Gamma \vdash E_1 : \tau' \rightarrow \tau \quad \Gamma \vdash E_2 : \tau'}{\Gamma \vdash E_1 E_2 : \tau}$

VAR  $\frac{}{x:\text{unit} \vdash x:\text{unit}}$       UNIT  $\frac{}{\emptyset \vdash () : \text{unit}}$   
 ABS  $\frac{}{\emptyset \vdash \lambda x:\text{unit}. x:\text{unit} \rightarrow \text{unit}}$   
 APP  $\frac{}{\emptyset \vdash (\lambda x:\text{unit}. x) () : \text{unit}}$

Type Safety:

If  $\emptyset \vdash E : \tau$  and  $E \xrightarrow{0 \text{ or more steps}} E'$  then either  $E'$  is a value or there exists  $E''$  such that  $E' \rightarrow E''$

2 parts:

"preservation": If  $\emptyset \vdash E : \tau$  and  $E \xrightarrow{*} E'$  then  $\emptyset \vdash E' : \tau$

"progress": If  $\emptyset \vdash E : \tau$  then either  $E$  is a value or there exists  $E'$  such that  $E \rightarrow E'$ .

$\emptyset \vdash E : \tau \xRightarrow{\text{prog}} E \rightarrow E' \xRightarrow{\text{pres}} \emptyset \vdash E' : \tau \xRightarrow{\text{prog}} E' \rightarrow E'' \xRightarrow{\text{pres}} \emptyset \vdash E'' : \tau \Rightarrow \dots$