CS440: Programming Languages and Translators

Lecture 14: Type Checking and Unification

Spring 2023

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Type checking isn't too hard

```
let rec sum (l: int list) : int =
  match (l: int list) with
  | ([]: int list) -> (0 : int)
  | (h::t : int list) -> ((h: int) + (sum t: int): int)
```

$$\frac{\dfrac{}{h: int} \quad \dfrac{\dfrac{}{sum: int\ list \rightarrow int} \quad \dfrac{}{t: int\ list}}{sum\ t : int}}{h + sum\ t : int}$$

Type checking isn't too hard... even if we only have inputs

```
let rec sum (l: int list) : ?? =
  match (l: ?? ) with
  | ([]: int list) -> (0 : ?? )
  | (h::t : int list) -> ((h: ?? ) + (sum t: ?? ): ?? )
```

$$\dfrac{\dfrac{}{h: int} \quad \dfrac{\dfrac{}{sum: int\ list \rightarrow int} \quad \dfrac{}{t: int\ list}}{sum\ t : int}}{h + sum\ t : int}$$

Type *inference* is a little harder

```
let rec sum l =
  match l with
  | [] -> 0
  | h::t -> h + sum t
```

Type *inference* is a little harder

```
let rec sum (l: [ ?? ]) =
  match l with
  | [] -> 0
  | h::t -> h + sum t
```

Type *inference* is a little harder

```
let rec sum (l: ?? ) =
  match (l: ?? ) with
  | [] -> 0
  | h::t -> h + sum t
```

Type *inference* is a little harder

```
let rec sum (l:    ??    ) =
  match (l:    ??    ) with
  | [] -> 0
  | h::t -> h + sum t
```

Type *inference* is a little harder

```
let rec sum (l: ??) : ?? =
  match (l: ??) with
  | [] -> (0 : ??)
  | h::t -> ((h: ??) + (sum (t: ??) : ??): ??)
```

Unification variables help us keep track of what we still have to figure out

- We'll use ?1, ?2, ?3, etc.
- Need to fill in the same type everywhere ?1 appears
- NOT the same as type variables 'a, 'b, etc., but difference is subtle

When we see something whose type we don’t know, add a unif. var.

```
let rec sum (l: ?1) : ?2 =
  match l with
  | [] -> 0
  | h::t -> h + sum t
```

Keep unification variables consistent

```
let rec sum (l: ?1) : ?2 =
  match (l: ?1) with
  | [] -> 0
  | h::t -> h + sum t
```

We can refine unification variables when we get more information

```
let rec sum (l: ?3 list) : ?2 =
  match (l: ?3 list) with
  | [] -> 0
  | h::t -> h + sum t
```

We can refine unification variables when we get more information

```
let rec sum (l: ?3 list) : ?2 =
  match (l: ?3 list) with
  | [] -> (0: ?2)
  | h::t -> ((h: ?3) + (sum (t: ?3 list): ?2): ?2)
```

We can refine unification variables when we get more information

```
let rec sum (l: ?3 list) : int =
  match (l: ?3 list) with
  | [] -> (0: int)
  | h::t -> ((h: ?3) + (sum (t: ?3 list): int): int)
```

We can refine unification variables when we get more information

```
let rec sum (l: int list) : int =
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  | [] -> (0: int)
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```

Unification

- Making types “look like” each other

- e.g., unify(?1, ?3 list)
- e.g., unify(?3, int)

What if you can’t unify?

```
let rec sum_bad (l: ?1) : ?2 =
  match (l: ?1) with
  | [] -> 0
  | h::t -> h +. sum t
```

What if you can't unify?

```
let rec sum_bad (l: ?3 list) : int =
  match (l: ?3 list) with
  | [] -> (0: int)
  | h::t -> (h +. sum t: int)
```

What if you can’t unify?

```
let rec sum_bad (l: ?3 list) : int =
  match (l: ?3 list) with
  | [] -> (0: int)
  | h::t -> ((h: ?3) +. (sum (t: ?3 list) : int): int)
```

What if you can't unify?

```
let rec sum_bad (l: float list) : int =
  match (l: float list) with
  | [] -> (0: int)
  | h::t -> ((h: float) +. (sum (t: float list) : int): int)
```

What if you can't unify?

```
let rec sum_bad (l: float list) : int =
  match (l: float list) with
  | [] -> (0: int)
  | h::t -> ((h: float) +. (sum (t: float list) : int): int)
```

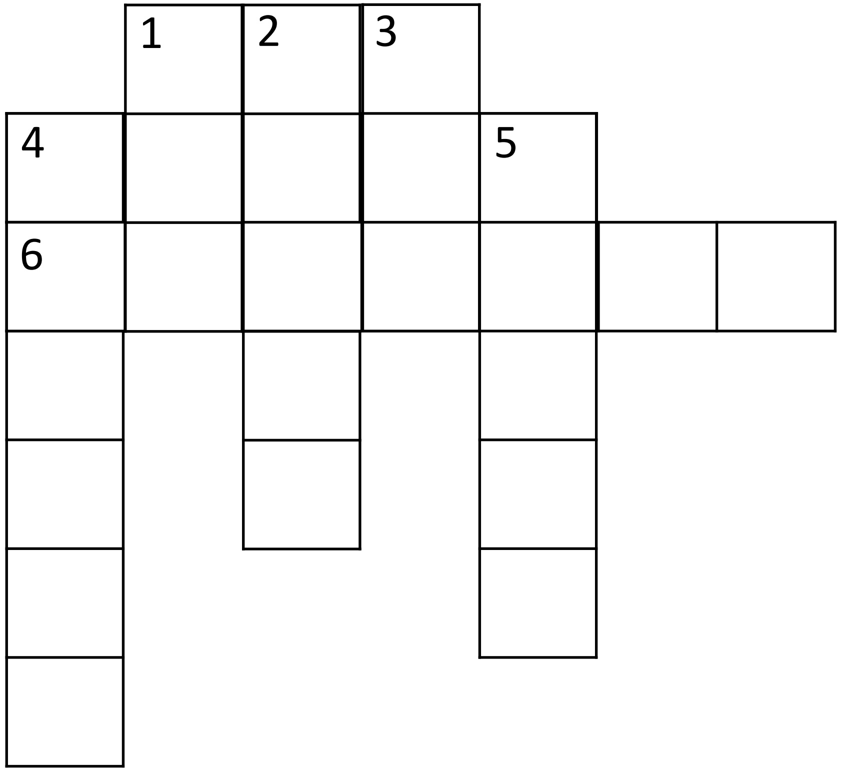
```
unify(float, int)
```

A: Type error

The goal of unification is to produce a *substitution* σ

- A mapping from unification variables to types
- e.g., [?1 -> ?3 list, ?2 -> int, ?3 -> int]

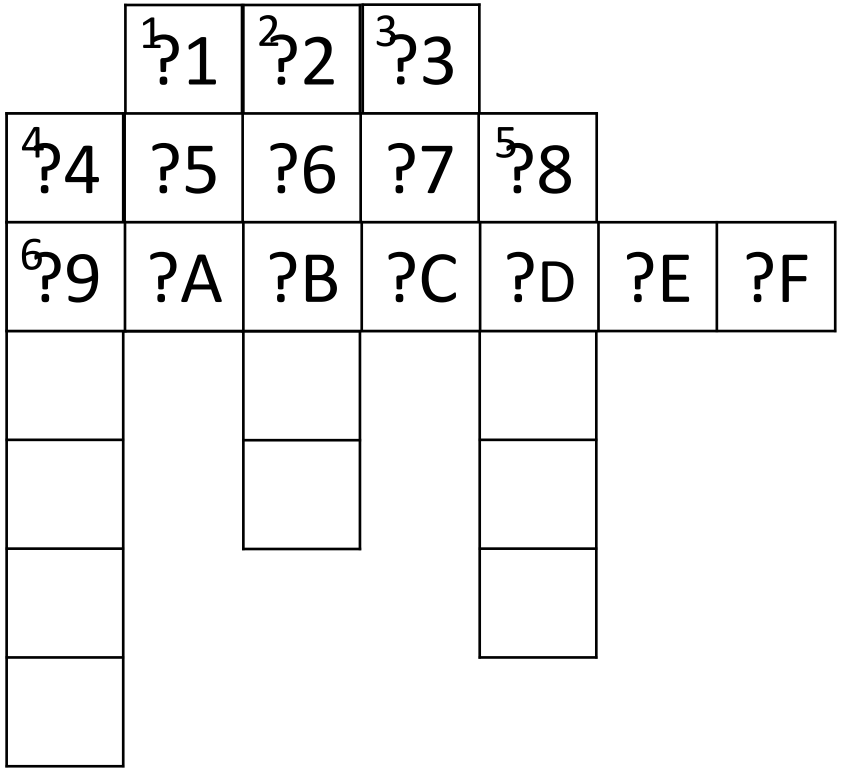
Unification example



Across
1. SB 104, SB 218E, e.g.
4. A functional language used at Jane Street Capital
6. [], h::t, (x, y), for example

Down
1. Maker of the Spectra 70 computer (abbr.)
2. ____ e with
3. ____ solver, which we hope we don't need for type inference
4. int ____, the type of "Some 42"
5. Xavier ____, inventor of 4-Across

Unification example



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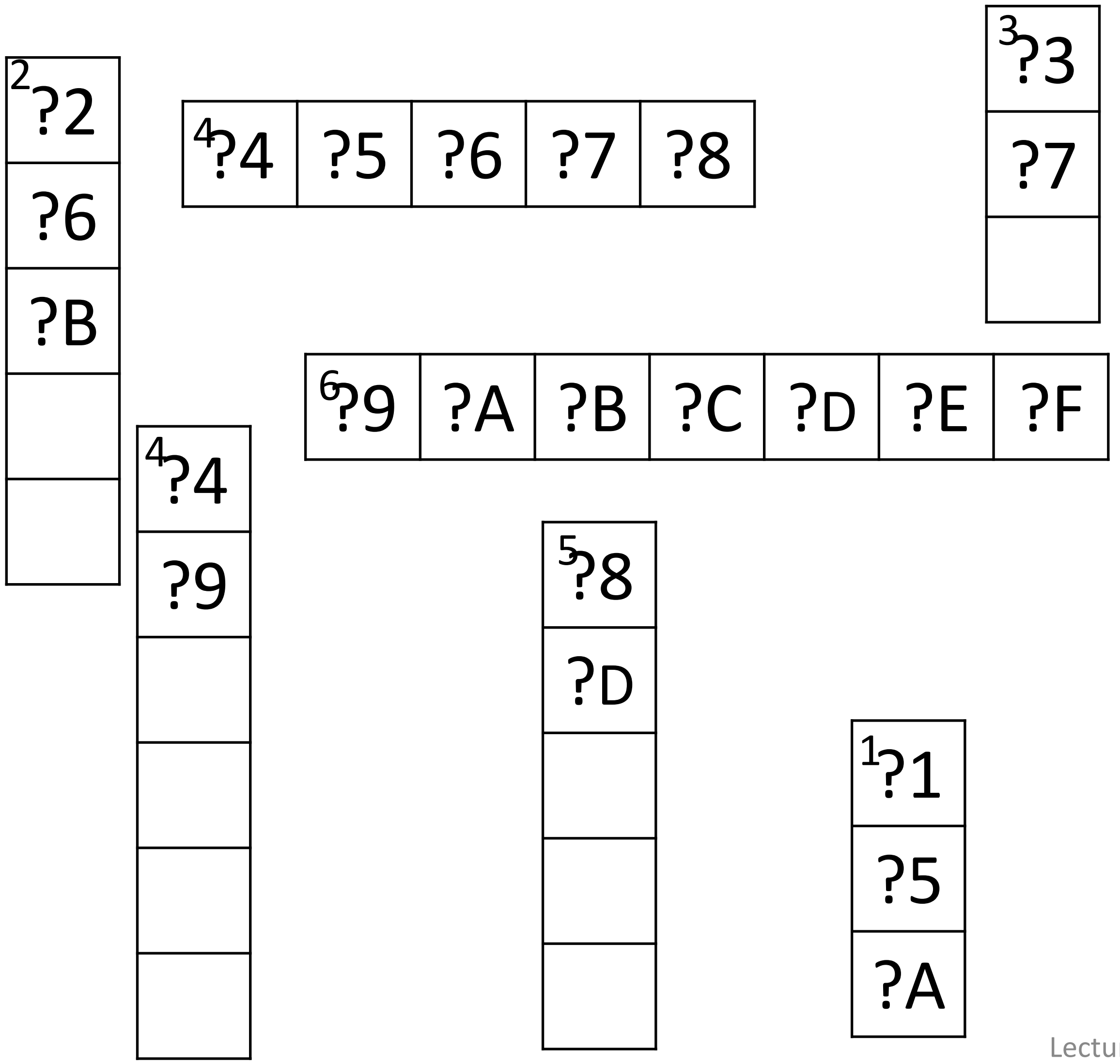
Unification example



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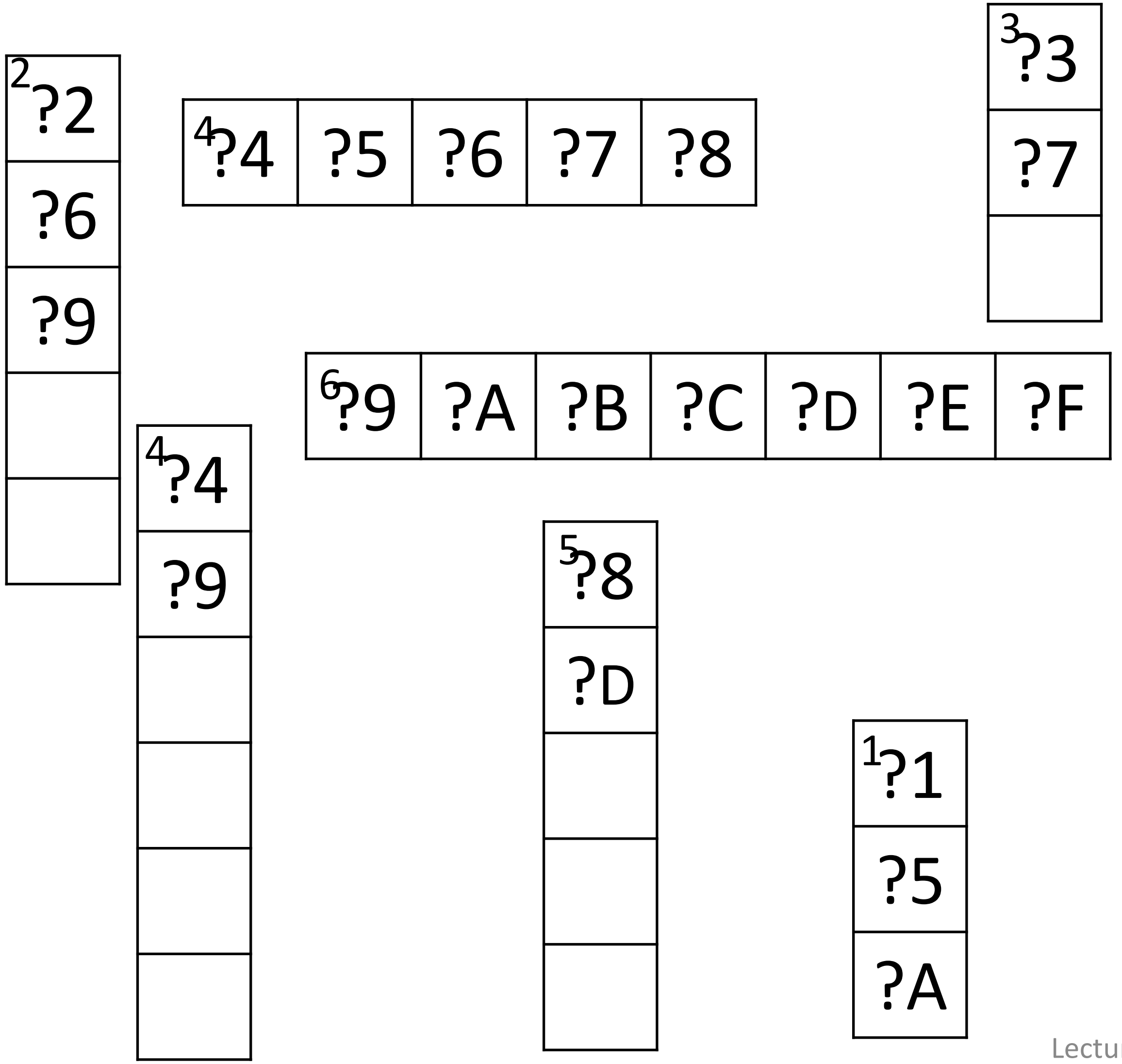
Unification example



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5. Xavier ____, inventor of 4-Across

?1 -> R, ?2 -> M, ?3 -> S, ?4 -> O, ?5 -> C, ?6 -> A,
?7 -> M, ?8 -> L, ?9 -> P, ?A -> A, ?B -> T, ?C -> T,
?D -> E, ?E -> R, ?F -> N

Substituting types

- We write $[\sigma]\tau$ to mean "τ with all of the substitutions in σ"
- [?1 -> int list, ?2 -> int](?1 -> ?2) = int list -> int

- "Simultaneous substitution": keep substituting until things don't change
  - [?1 -> ?3 list, ?2 -> int, ?3 -> int](?1 -> ?2) = int list -> int

We build up a substitution as we unify

```
let rec sum l =
  match l with
  | [] -> 0
  | h::t -> h + sum t
```

We build up a substitution as we unify

```
let rec sum (l: ?1) : ?2 =
  match l with
  | [] -> 0
  | h::t -> h + sum t
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We build up a substitution as we unify

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let rec sum (l: ?1) : ?2 =
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let rec sum (l: ?1) : ?2 =
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```

?1 -> ?3 list,

We build up a substitution as we unify

```
let rec sum (l: ?1) : ?2 =
  match (l: ?1) with
  | [] -> (0: ?2)
  | h::t -> h + sum t
```

?1 -> ?3 list,
?2 -> int

We build up a substitution as we unify

```
let rec sum (l: ?1) : ?2 =
  match (l: ?1) with
  | [] -> (0: ?2)
  | h::t -> (h: ?3) + (sum (t: ?3 list) : ?2) : ?2
```

?1 -> ?3 list,
?2 -> int

We build up a substitution as we unify

```
let rec sum (l: ?1) : ?2 =
  match (l: ?1) with
  | [] -> (0: ?2)
  | h::t -> (h: ?3) + (sum (t: ?3 list) : ?2) : ?2
```

?1 -> ?3 list,

?2 -> int

?3 -> int

Q: What if we have leftover unification variables when we're done?

```
let rec length (l: ?1) : ?2 =
  match (l: ?1) with
  | [] -> (0: ?2)
  | h::t -> (1: ?4) + (length (t: ?3 list) : ?2) : ?2
```

?1 -> ?3 list,
?2 -> int
?4 -> int

Q: What if we have leftover unification variables when we're done?

```
let rec length (l: ?1) : ?2 =
  match (l: ?1) with
  | [] -> (0: ?2)
  | h::t -> (1: ?4) + (length (t: ?3 list) : ?2) : ?2
```

?1 -> ?3 list,

?2 -> int

?4 -> int

?3 -> 'a

A: They become type variables (but it's a little complicated)