Written Theory Qualifier Exam

Your number: _____

Time limit: 2.5 hours. Use only the notes supplied by the Department

SPRING 2019, CS DEPARTMENT, IIT

For every question, please write your answer in a clean and concise way. Use additional pages, start a new page with each problem and write only on one side of the paper.

Use procedures if you want – – marking clearly what the parameters are and what they do, and with what running time in terms of its parameters. Unless the procedures are from the textbook, write pseudocode for the procedures. You should be given a copy of this textbook.

Problem 1. Prove that, if a node in a binary-search-tree has two children, its succesor has no left child and its predecessor no right-child.

Problem 2. Most graph algorithms that take an adjacency-matrix representation as input require time $\Omega(|V|^2)$, but there are some exceptions. Show how to determine whether a directed graph G contains a universal sink a vertex with in-degree $|V| - 1$ and out-degree 0 in time $O(V)$, given an adjacency matrix for G.

Pseudocode is required for this problem. Also discuss correctness and analyze the running time.

Problem 3. KNAPSACK is the following problem: Given integers K and B, and items $\{1, 2, \ldots, n\}$ each with integer size s_i and integer profit p_i, find a subset $D \subseteq \{1, 2, \ldots, n\}$ such that $\sum_{i \in D} s_i \leq B$ and $\sum_{i \in D} p_i \geq K$. As an aside, the common optimization problem is: we attempt to maximize K, the total profit, with a bound B on the total size of items one can pick (the set D). All the numbers are written in binary.

Prove that *Knapsack*, as a language or decision problem, is NP-hard. The reduction must come from a problem proved NP-hard in the textbook.