Written Theory Qualifier Exam

Your number: _____

Time limit: 2.5 hours. Use only the notes supplied by the Department

FALL 2019, CS DEPARTMENT, IIT

For every question, please write your answer in a clean and concise way. Use additional pages, start a new page with each problem and write only on one side of the paper.

Use procedures if you want – – marking clearly what the parameters are and what they do, and with what running time in terms of its parameters. Unless the procedures are from the textbook, write pseudocode for the procedures. You should be given a copy of this textbook.

Problem 1. Suppose you are given k n-element sorted sequences A_i, each representing a set (none has duplicate entries). Describe an $O(nk \log k)$-time method for computing a sorted sequence representing the set $\cup_{i=1}^{k} A_i$ (with no duplicates).

Pseudocode is required. You do not have to argue correctness (but, obviously, your method must be correct), but must justify the running time.

Problem 2. This problem refers to the DFS(G) and DFS-Visit(u) procedures from the textbook, applied to an **undirected** graph. Show how to modify depth-first search so that it assigns to each vertex v an integer label $v.cc$ between 1 and k, where k is the number of connected components of G, such that $u.cc = v.cc$ if and only if u and v are in the same connected component of the input **undirected** graph.

Write the pseudocode of your modified DFS(G) and DFS-Visit(u). Argue that your modifications do not increase the running time of the algorithm by more than a constant factor.

Problem 3. Suppose you have one machine and a set of n jobs $a_1, a_2, \ldots, a_n$ to process on that machine. Each job a_j has a processing time t_j, a profit p_j, and a deadline d_j. The machine can process only one job at a time, and job a_j must run uninterruptedly for t_j consecutive time units. If job a_j is completed by its deadline d_j, you receive a profit p_j, but if it is completed after its deadline, you receive a profit of 0. Give a polynomial-time algorithm to find the schedule that obtains the maximum amount of profit, assuming that all processing times are integers between 1 and n.

It may be helpful (and it is OK here) to assume that the jobs are sorted by increasing order of deadlines.

Present pseudocode, discuss correctness, and analyze the running time of your algorithm.