

# Written Theory Qualifier Exam

Your number: \_\_\_\_\_

**Time limit: 2.5 hours. Use only the notes supplied by the Department**

SPRING 2018, CS DEPARTMENT, IIT

For every question, please write your answer in a clean and concise way. Use additional pages, start a new page with each problem and write only on one side of the paper.

Use procedures if you want -- marking clearly what the parameters are and what they do, and with what running time in terms of its parameters. Unless the procedures are from the textbook, write pseudocode for the procedures. You should be given a copy of this textbook. Using procedures from the textbook is recommended so that you do not have to spend too much time writing.

**Problem 1.** Consider the problem of determining whether an arbitrary sequence  $\langle x_1, x_2, \dots, x_n \rangle$  of  $n$  numbers contains repeated occurrences of some number. Show that this can be done in  $O(n \log n)$  time, where  $\log n$  stands for  $\log_2 n$ .

Pseudocode is required. Do analyze the running time.

**Problem 2.** We say that a digraph  $G = (V, E)$  is *half-connected* iff for all  $u, v \in V$ , there exists either a directed path from  $u$  to  $v$  or a directed path from  $v$  to  $u$ . Give an  $O(|V| + |E|)$ -time and space algorithm to determine if a given digraph (adjacency lists representation) is half-connected.

Pseudocode is required. Do analyze the running time and do prove that your algorithm is correct.

**Problem 3.** Suppose we wish not only to increment a binary number, but also to reset it to zero (i.e., make all bits in it 0). Counting the cost to examine or modify a bit as 1, show how to implement a binary number as an array of bits so that any sequence of  $n$  INCREMENT and RESET operations costs  $O(n)$  on an initially zero number. Do analyze the running time.

**Hint:** Keep a pointer to the high-order 1.