Written Theory Qualifier Exam

Your number: ____________________

Time limit: 2.5 hours. Use only the notes supplied by the Department

Fall 2017, CS Department, IIT

For every question, please write your answer in a clean and concise way. Use additional pages, start a new page with each problem and write only on one side of the paper.

Use procedures if you want – – marking clearly what the parameters are and what they do, and with what running time in terms of its parameters. Unless the procedures are from the textbook, write pseudocode for the procedures. You should be given a copy of this textbook.

Problem 1. Augment the **stack** data structure to do in constant time, besides push/insert and pop/remove, findmin. Describe the data structure, and present pseudocode for push/insert, pop/remove, and findmin. Argue that each operation takes constant time. Argue that findmin correctly returns the element with minimum key.

Problem 2. Describe an efficient greedy algorithm for making change for a specified value using a minimum number of coins, assuming there are four denominations of coins (called dimes, nickels, quarters, and pennies), with values 10, 5, 25, and 1, respectively.

The input of your algorithm is an integer number N representing the total number of cents. The output specifies for each denomination how many coins are to be used. Here, "efficient" means $O(N)$.

1. Present pseudocode, running time analysis, proof of correctness.
2. Find four denominations of coins, one of which is 1c, and one value of N for which the greedy algorithm fails to return the minimum number of coins. Show the greedy and optimum solutions.
3. Present a dynamic program for any set of k denominations. Strive for running time of $O(kN)$, but make sure that the running time is polynomial in k and N. Present the pseudocode, discuss correctness, and analyze the running time.

Problem 3. A **multiple source-sink network** is a tuple $G = (V, E, c, S, T)$, where V is a set of vertices, E is a set of directed edges (parallel edges are allowed), $S \subset V$ is the set of **sources**, and $T \subset V$ is the set of **sinks**, c is a **capacity** function: $c : E \to Z_+$. Also, $S \cap T = \emptyset$. That is, sources are distinct from sinks.

A function $f : E \to R_+$ is called a *flow* if the following three conditions are satisfied:

1. *conservation of flow at interior vertices:* for all vertices u not in $S \cup T$,

$$\sum_{e \in \delta^-(u)} f(e) = \sum_{e \in \delta^+(u)} f(e) \ ;$$

2. *capacity constraints:* $f \leq c$ pointwise: i.e. for all $e \in E$,

$$f(e) \leq c(e) \ .$$

Assume that non-negative quantities p_s, for $s \in S$, and q_t, for $t \in T$, are given. The goal of this problem is to determine if a valid flow exists such that for all $s \in S$:

$$\sum_{e \in \delta^+(s)} f(e) - \sum_{e \in \delta^-(s)} f(e) = p_s$$

(in words, the source s "produces" p_s units of flow) and such that for all $t \in T$:

$$\sum_{e \in \delta^-(t)} f(e) - \sum_{e \in \delta^+(t)} f(e) = q_t$$

(in words, the sink t "consumes" q_t units of flow).

Use Network Flows to give a polynomial-time algorithm for this **decision** problem (the answer is YES or NO).