

# Written Theory Qualifier Exam

Your number: \_\_\_\_\_

**Time limit: 2.5 hours. Use only the notes supplied by the Department**

SPRING 2016, CS DEPARTMENT, IIT

For every question, please write your answer in a clean and concise way. Use additional pages, start a new page with each problem and write only on one side of the paper.

Use procedures if you want – marking clearly what the parameters are and what they do, and with what running time in terms of its parameters. Unless the procedures are from the textbook, write pseudocode for the procedures. You should be given a copy of this textbook.

## **Problem 1.**

This problem refers to binary max-heaps as defined in the Chapter 6 of the textbook, and the operations described and implemented there.

We want to claim an amortized cost of  $O(1)$  for EXTRACT-MAX, which is actually possible with an amortized cost of  $O(\lg n)$  for INSERT. Find a potential function  $\Phi$  to obtain these bounds. Then prove the bounds (do not change the implementation from the textbook, so no pseudocode is needed here).

**Problem 2.**

In the **art gallery guarding** problem we are given a line  $L$  that represent a long hallway in an art gallery. We are also given a set  $X = \{x_0, x_1, \dots, x_{n-1}\}$  of real numbers that specify the position of paintings in this hallway. Suppose a single guard can protect all the paintings within distance 1 of his position (on both sides).

Design an algorithm for finding a placement of guards that uses the minimum number of guards to protect all the paintings with positions in  $X$ . Present the pseudocode. Prove that your algorithm minimizes the number of guards. Analyze its running time.  $O(n \log n)$ -time is needed for full marks; polynomial running time gives 80% of the marks, and exponential (provided it does give the correct answer) only 40%.

**Problem 3.** Let  $G = (V, E, c, s, t)$  be a flow network with integer capacities, and let  $f$  be an integral maximum flow in  $G$ . Let  $G' = (V, E, c', s, t)$  differ from  $G$  on one single edge  $e$ :  $c'(e) = c(e) - 1$ . Give a  $O(|V| + |E|)$ -time algorithm to obtain a maximum flow  $f'$  in  $G'$ .

Present a higher-level pseudocode (with clear English descriptions, i.e. "construct the residual network  $G_f$ "), argue correctness, and do the running-time analysis.