

## Written Theory Qualifier Exam

Your number: \_\_\_\_\_

**Time limit: 2.5 hours. Use only the notes supplied by the Department**

FALL 2011, CS DEPARTMENT, IIT

For every question, please write your answer in a clean and concise way. Use additional pages, start a new page with each problem and write only on one side of the paper.

Use procedures if you want – marking clearly what the parameters are and what they do, and with what running time in terms of its parameters. Unless the procedures are from the textbooks, write pseudocode for the procedures.

1. You are given a binary tree  $T$  represented by the following data structure. Nodes have three fields: element, left-child, and right-child. You are given the location (or pointer to) the root. Let  $|T|$  be the number of nodes in  $T$ .

Given tree nodes  $x$  and  $y$  such that  $y$  is a child of  $x$ , removing from  $T$  the link from  $x$  to  $y$  splits  $T$  into two trees, which we call  $T'(y)$  (with the same root as  $T$ ) and  $T_y$  (with the root  $y$ ). The goal is to find such  $x, y$  such that  $|T'(y)|$  and  $|T_y|$  are as close as possible (that is, minimize the absolute value of  $|T'(y)| - |T_y|$ ).

Write pseudocode to find the location of (or pointer to)  $x, y$ , with the running time  $O(|T|)$ . Argue this is indeed the running time. Prove that parent-child nodes  $x, y$  always exist such that the biggest of  $|T'(y)|$  and  $|T_y|$  is at most  $3|T|/4$ .

2. Consider the following divide-and-conquer algorithm for computing minimum spanning trees. Given a graph  $G = (V, E)$ , partition the set  $V$  of vertices into two sets  $V_1$  and  $V_2$  such that  $|V_1|$  and  $|V_2|$  differ by at most 1. Let  $E_1$  be the set of edges that are incident only on vertices in  $V_1$ , and let  $E_2$  be the set of edges that are incident only on vertices in  $V_2$ . Recursively solve a minimum spanning tree problem on each of the two subgraphs  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$ . Finally, select the minimum-weight edge in  $e$  that crosses the cut  $(V_1, V_2)$ , and use this edge to unite the resulting two minimum spanning trees into a single spanning tree.

Either argue the algorithm correctly computes a minimum spanning tree of  $G$ , or provide an example for which the algorithm fails, showing the run of the algorithm and a better spanning tree.

3. Classify the following language appropriately from amongst the following categories: (Regular languages, Context Free languages, Turing-recognizable languages). Give proofs:

(a)  $\{0^n 1^m 0^p 1^r \mid n, m, p, r \geq 0 \wedge n = 2m \wedge r = 2p\}$ .

4. Show that the class of Turing-recognizable languages is closed under the operation of concatenation. That is, show that if  $M_1$  and  $M_2$  are two (not necessarily halting) Turing machines, then there exists a Turing machine  $M$  such that  $L(M) = L(M_1) \circ L(M_2)$ . Prove the set equality, and be aware of infinite loops.