

# Theory Qualifier Exam

Your number: \_\_\_\_\_

**Time limit: 2.5 hours. Use only the notes supplied by the Department**

SPRING 2010, CS DEPARTMENT, IIT

For every question, please write your answer in a clean and concise way. Use additional pages, start a new page with each problem and write only one side of the paper.

If you are asked to write an algorithm for a question, you have to write the **pseudo-code** of your algorithm and also put explanations about your pseudo-code. Also show correctness and estimate the running time.

Use procedures if you want – marking clearly what the parameters are and what they do, and with what running time in terms of its parameters. Unless the procedures are from the textbooks, write pseudocode for the procedures.

1. Present the modified pseudocode for Depth-First Search so that it prints out every edge in directed graph  $G$ , together with its type (forward, back, tree, cross).
2. We are given two arrays of size  $n$   $A$  and  $B$ , of integers, Consider the  $n$  closed intervals  $[A[i], B[i]]$ , which may not be disjoint. The union of these  $n$  intervals is another set of intervals, which can be made disjoint and maximal. Present an  $O(n \log n)$  algorithm to output this new set of disjoint and maximal intervals into arrays  $C$  and  $D$ .
3. Show that the problem of testing whether a CFG generates some string in  $1^*$  is decidable. In other words, show that  $\{\langle G \rangle \mid G \text{ is a CFG over } \{0, 1\}^* \text{ and } 1^* \cap L(G) \neq \emptyset\}$  is a decidable language.
4. Prove that the following languages over  $\{0, 1, 2\}$  are not regular:
  - (a)  $\{0^n 1^n 2^n \mid n \geq 0\}$
  - (b)  $\{w \mid w \neq w^R\}$