

PhD Qualifier Exam for Spring 2008 — Theory Area

CS DEPARTMENT, IIT

Your number: _____

Time limit: 2.5 hours. Use only the books supplied by the Department

There are 3 questions in this exam. For every question, please write your answer in a clean and concise way. Use additional pages, start a new page with each problem and write only one side of the paper.

If you are asked to write an algorithm for a question, you have to write the **pseudo-code** of your algorithm and also put explanations about your pseudo-code. Also show correctness and estimate the running time. Every statement must be proven (or be obvious to the grader).

Problem 1 A jogger wants to follow the least undesirable cycle of roads starting at her home. Each road has an “index of undesirability” and can be traversed in either direction; the jogger must follow a nonempty cycle of roads and no road can be used twice. Formulated as a graph problem, the jogger has an undirected weighted graph $G = (V, E)$, and must determine the nonempty cycle of minimum weight starting (and ending) at vertex s .

1. Show how to use multiple applications of Dijkstra’s shortest path algorithm to obtain the optimum jogger’s route in time $O(|V|^2 \log |V| + |E||V|)$.

Be precise: each time you want to use Dijkstra’s explain which graph is the input of the algorithm,

2. Let T be the shortest path tree constructed by Dijkstra’s shortest path algorithm for starting vertex s in G . Prove that some optimum jogger’s route has all but one of its edges in T , and furthermore, that s is the lowest common ancestor in T of the end points of that edge.

3. Use the result in part (b) to modify Dijkstra’s shortest path algorithm so it finds an optimum jogger’s route in time $O(|V| \log |V| + |E|)$.

Write pseudocode, discuss the running time and correctness.

4. Prove that the *masochistic jogger’s problem*, to find a route of maximum undesirability, is NP-hard. Here you can use any of the know NP-hard problems from the textbooks.

Problem 2 (a) Classify the following language appropriately from amongst the following categories: (Regular Languages, Context Free languages, Turing-recognizable languages, non-Turing-recognizable languages). Give proofs:

(i) $L = \{ww^Rw \mid w \in \{0, 1\}^*\}$, where w^R is obtained by reversing the characters in w

(ii) $L = \{w \mid w \in \{0, 1\}^* \text{ and } w \text{ contains at least three 0's and at most one 1}\}$

Problem 3 Given a sequence of numbers a_1, a_2, \dots, a_n (some of them might be negative) stored in an array, give an algorithm to find two indices $i \leq j$ such that $\sum_{k=i}^j a_k$ is maximum, among all the pairs $1 \leq i \leq j \leq n$. Your algorithm **must** run in $O(n)$ time.

Pseudocode, correctness, running-time analysis.