

THEORY-QUALIFIER:FALL 2005

2.5 Hours

Max Marks 100

1. A network is called *unreliable* if there exists an edge the removal of which disconnects the graph. The network is *d-unreliable* if there exists a node  $u$  such that a fault at any one of the edges incident to it or at  $u$  disconnects the graph. Design an algorithm to find if the connected graph is unreliable or d-unreliable. Only a linear-time algorithm will receive full credit.

You have to write the **pseudo-code** of your algorithm and also put explanations about your pseudo-code. You can use procedures from the book chapter inside your algorithm - make sure to specify the parameters. Analyze the running time and prove the correctness of your algorithm. You can use any theorem proven in the book (chapter).

2. (a) Classify the following language appropriately from amongst the following categories: (Regular Languages, Context Free languages, Turing-recognizable languages). Give proofs:
  - (i)  $L = \{w | w = w_1 w_1^r, w \bmod 4 = 0, w \in \{0, 1\}^*\}$
  - (ii)  $L = \{w | w = w_1 w_1, w \bmod 4 = 0, w \in \{0, 1\}^*\}$
  - (iii)  $L = \{w | w \bmod 4 = 0, w \in \{0, 1\}^*\}$

3. Consider a clique,  $C$ , in a graph  $G$ . What kind of structure is formed on the set  $C$  in the complement of  $G$ ?

Given that the Clique problem, i.e. the problem of finding a clique of size  $k$ , is NP-Hard (by a reduction from 3-SAT) show that the Independent set (IS), the problem of finding an independent set of size  $l$ , is NP-Hard.

Show that there exists a polynomial time reduction from IS to 3-SAT.