Illinois Institute of Technology Department of Computer Science

Qualifying Exam – Languages

Fall 2015

This is a **closed book** and **closed notes** exam.
Do **ALL** problems in this booklet. Read each question very carefully.
You may detach pages, but **you must return all pages of this exam.**

Last Four Digits of CWID: Axxx

Problem	Points	Score
Total	0	
Percent	100	

Part I: CS440

Question 1) (15 points) Please do NOT use more than three sentences to answer each of the following questions. We will NOT grade anything after the third sentence in your answer sheet.

1. Many languages are not compiled to machine code, but instead compiled to a byte code. Examples include Python and Java.

   (a) What advantages does this have?

   Solution:

   Portability, since a the same program will run on any machine that has a byte-code interpreter. They could also mention flexibility, since it is often easier to compile to a byte code than to machine code.

   (b) How can such environments be performant?

   Solution:

   Many byte code interpreters contain just-in-time compilers or native code compilers that further compile the byte code to machine code.

2.

   (a) What is a closure?

   Solution:

   A closure is a function together with the definitions of any free variables (i.e., the environment) at the time the closure was created.

   (b) What advantages does it give a language to have them?

   Solution:

   Closures allow for higher order functions and can express certain concepts very tersely. They are similar in power to objects.

3. We can divide languages roughly into two classes: statically typed (e.g., Java, Haskell, C++) and dynamically typed (e.g., Lisp, Clojure, Ruby). Explain the tradeoffs between these two systems.

   Solution:

   Statically typed languages are more restrictive in the kinds of programs that can be written, but they also allow the compiler to catch more errors than dynamically typed languages, and also allow the compiler to generate more optimal code in most cases. It is not the case that statically typed languages require type declarations, but most of them do.

Question 2) (6 points) For the three subquestions, incorrect answer will have -1 point penalty.

1. Dose 01001 belong to the language represented by $(0+1)^*$?

   Solution:

   Yes

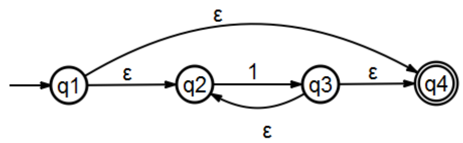
2. Dose 01010 belong to the language represented by $(0+1)^*$?

   Solution:

   yes

3. Dose 01010 belong to the language represented by $(0+1)^*00(0+1)^*$?

   Solution:



No

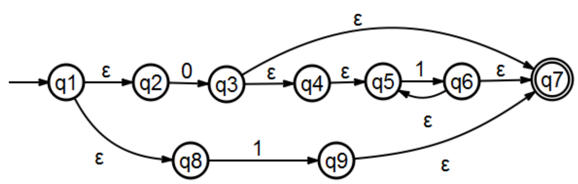
Question 3) (6 points) Give the regular expression for the following two automata.

1. The alphabet is $\Sigma = \{1\}$.

   Solution:

   1^*

2. The alphabet is $\Sigma = \{0, 1\}$.



   Solution:

   $\{01^* + 1\}$

Part II: CS536

Question 4) (5 points) Let $W \equiv$ **while** $x > 0$ **do** S **od** , where $S \equiv x := x - 1; y := y/2$, both x and y are integers. For /, use integer division with truncation. Let $\sigma_0 = \{x = 3, y = 32\}$, calculate $\mathcal{M}[\![W]\!](\sigma_0)$. Show your work steps.

Solution:

$$\mathcal{M}[\![W]\!](\sigma_0) = \{x = 0, y = 4\}$$

Question 5) (8 points) Let $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ be the set of integers, and $x \in \mathbb{Z}$. For following four question, if the triple is invalid, give a counter example state (a state in which the triple is unsatisfied).

1. Is the triple $\{T\}$ **while** $x < 0$ **do** $x := x + 1$ **od** $\{x \geq 0\}$ valid under partial correctness?

   Solution:

   Yes

2. Is the triple $\{T\}$ **while** $x < 0$ **do** $x := x + 1$ **od** $\{x \geq 0\}$ valid under total correctness?

   Solution:

   Yes

3. Is the triple $\{T\}$ **while** $x \neq 0$ **do** $x := x + 1$ **od** $\{x \geq 0\}$ valid under partial correctness?

   Solution:

   Yes

4. Is the triple $\{T\}$ **while** $x \neq 0$ **do** $x := x + 1$ **od** $\{x \geq 0\}$ valid under total correctness?

   Solution:

   No, $\sigma = \{x = 1\}$

Question 6) (5 points) Calculate $\texttt{wp}(S,\ y < z)$, where $S \equiv$ **if** $y = 0$ **then** $y := 1$ **else** $y := y * z; z := z + 1$ **fi** . Show your work steps.

Solution:

$$\texttt{wp}(S,\ y < z) \equiv (y = 0 \wedge 1 < z) \vee (y \neq 0 \wedge y * z < z + 1)$$

Question 7) (5 points) Calculate $\texttt{sp}(y < z,\ S)$, where $S \equiv$ **if** $y = 0$ **then** $y := 1$ **else** $y := y * z; z := z + 1$ **fi** . Show your work steps.

Solution:

$$\texttt{sp}(y < z,\ S) \equiv (y_0 < z \wedge y_0 = 0 \wedge y = 1) \vee (y_0 < z_0 \wedge y_0 \neq 0 \wedge y = y_0 * z_0 \wedge z = z_0 + 1)$$

Question 8) (6 points) Suppose $\sigma \models \texttt{wlp}(S,\ q)$. If we run S starting in state σ, can we diverge? If we converge to a final state τ, do we know whether $\tau \models q$ or $\tau \models \neg q$?

Solution:

Yes, it may diverge.

If converge, then $\tau \models q$.

Question 9) (10 points) Calculate p below so that it is the weakest precondition of the assignment. Simplify p using logical manipulations to maximize its readability. You can assume that x, y, and $y+1$ are legal indexes for array b (so you can omit those tests from p). Show your work steps.

$$\{p\}b[x] := b[y]\{b[x] \le b[y] < b[y+1]\}$$

Solution:

$$\begin{aligned}
p &\equiv (b[x] \le b[y] < b[y+1])[b[x] := b[y]] \\
&\equiv b[x][b[x] := b[y]] \le b[y][b[x] := b[y]] < b[y+1][b[x] := b[y]] \\
&\equiv b[y] \le \ \textbf{if} \ \ x = y \ \ \textbf{then} \ \ b[y] \ \ \textbf{else} \ \ b[y] \ \ \textbf{fi} \ < \ \textbf{if} \ \ x = y+1 \ \ \textbf{then} \ \ b[y] \ \ \textbf{else} \ \ b[y+1] \ \ \textbf{fi} \\
&\equiv b[y] \le b[y] < \ \textbf{if} \ \ x = y+1 \ \ \textbf{then} \ \ b[y] \ \ \textbf{else} \ \ b[y+1] \ \ \textbf{fi} \\
&\equiv \ \textbf{if} \ \ x = y+1 \ \ \textbf{then} \ \ b[y] \le b[y] < b[y] \ \ \textbf{else} \ \ b[y] \le b[y] < b[y+1] \ \ \textbf{fi} \\
&\equiv \ \textbf{if} \ \ x = y+1 \ \ \textbf{then} \ \ \texttt{false} \ \ \textbf{else} \ \ b[y] < b[y+1] \ \ \textbf{fi} \\
&\equiv (x = y+1 \wedge \texttt{false}) \vee (x \ne y+1 \wedge b[y] < b[y+1]) \\
&\equiv x \ne y+1 \wedge b[y] < b[y+1]
\end{aligned}$$

Question 10) (10 points) Given a Hoare Triple as follows:

$$\{n \ge 1\}S\{1 \le 2\hat{\ }r \le n < 2\hat{\ }(r+1)\}$$

where

$$\begin{aligned}
S \equiv\ & r := 0;\ s := n; \\
& \textbf{while}\ s \ne 1\ \ \textbf{do} \\
& \qquad r := r+1;\ s := s/2 \\
& \textbf{od}
\end{aligned}$$

In the triple above, ˆmeans exponentiation and / is integer division with truncation. Find an invariant and a bound function for the loop and give a full proof outline for total correctness for the program.

Solution:

$\{n \geq 1\}$
$r := 0; s := n;$
$\{\textbf{inv} : 1 \leq s * 2\hat{\ }r \leq n < s * 2\hat{\ }(r+1)\}$
$\{\textbf{bd} : n - 2\hat{\ }r\}$
while $s \neq 1$ **do**
    $\{1 \leq s * 2\hat{\ }r \leq n < s * 2\hat{\ }(r+1) \wedge s \neq 1\}$
    $r := r + 1;\ s := s/2$
    $\{1 \leq s * 2\hat{\ }r \leq n < s * 2\hat{\ }(r+1)\}$
od
$\{1 \leq s * 2\hat{\ }r \leq n < s * 2\hat{\ }(r+1) \wedge s = 1\}$
$\{1 \leq 2\hat{\ }r \leq n < 2\hat{\ }(r+1)\}$

Multiple Choice Answer Sheet

Each question has exactly one correct answer. If you think more than one answer is true, pick the one that is most true.

Select your choice by circling the corresponding letter. If you make a mistake, draw an "X" through the choice. If you really mess it up, cross them all out and draw a box clearly labeled with your answer. In the event that your answer is hard to read, all reasonable interpretations will be used. For example, a letter that looks like both an 'a' and a 'd' will be considered both. So be neat.

11	(a)	(b)	(c)	(d)
12	(a)	(b)	(c)	(d)
13	(a)	(b)	(c)	(d)
14	(a)	(b)	(c)	(d)
15	(a)	(b)	(c)	(d)
16	(a)	(b)	(c)	(d)

Question 11) (4 points) Which of the following is equivalent to $(\neg a \vee \neg b)$?

a) $\neg a \wedge \neg b$

b) $\neg a \vee b$

c) $a \rightarrow \neg b$

d) $\neg b \rightarrow a$

Solution:

c

Question 12) (4 points) Suppose we have a postcondition $R \equiv A \wedge C$. Which of the following would be a valid candidate for a corresponding loop invariant?

a) $A \wedge C \vee B$

b) $A \wedge C$

c) $B \wedge R$

d) $C \rightarrow R$

Solution:

a

Question 13) (4 points) Consider the disjoint parallel program $[S_1; S_3 \parallel S_2; S_4]$. Which of the following is an equivalent sequential program?

a) $S_3; S_1; S_4; S_2$

b) $S_1; S_3; S_4; S_2$

c) $S_3; S_4; S_1; S_2$

d) $S_2; S_4; S_1; S_3$

Solution:

d

Question 14) (4 points) Which of the following pairs of Hoare triples demonstrates interference freedom?

a) $S_1 \equiv \{s = 0\} s := s + x \{s = x\}$ and $S_2 \equiv \{s = 0\} s := s + y \{s = y\}$

b) $S_1 \equiv \{x = y\} x := x + 1 \{x = y + 1\}$ and $S_2 \equiv \{x = y\} y := y + 1 \{x + 1 = y\}$

c) $S_1 \equiv \{s = 0 \vee s = y\} s := s + x \{s = x + y\}$ and
$S_2 \equiv \{s = 0 \vee s = x\} s := s + y \{s = x + y\}$

d) $S_1 \equiv \{x = 0\} x := x + 1 \{x = 1 \vee x = 2\}$ and
$S_2 \equiv \{y = 0\} y := y + 1 \{y = 1 \vee y = 2\}$

Solution:

d

Question 15) (4 points) Consider the following loop.

$$\begin{array}{rl} S \equiv & \textbf{do}\ \ 3 < i < 10 \rightarrow i := i + 1 \\ & \ \ \Box\ 0 < i < 7 \rightarrow i := i - 1 \\ & \textbf{od} \end{array}$$

Which of the following is a possible result for this program?

a) $i = 12$

b) Δ

c) $\perp$

d) $fail$

Solution:

c

Question 16) (4 points) Consider the following nondeterministic program.

$$\begin{array}{rl} S \equiv & \textbf{if}\ \ i < 10 \rightarrow i := i + 1 \\ & \ \ \Box\ j > 10 \rightarrow j := j - 1 \\ & \textbf{fi} \end{array}$$

Give a state that is guaranteed to cause the program to exit with state $fail$.

a) $i = j$

b) $i = 10 \wedge j = 10$

c) $i + j = 20$

d) No states cause failure.

Solution:

b

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